Optimization techniques for a model problem of saltwater intrusion in coastal aquifers

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- *Part 2.* Mathematical approach-model equations
- *Part 3.* Types of coastal freshwater aquifers
- *Part 4.* Pumping optimization methods
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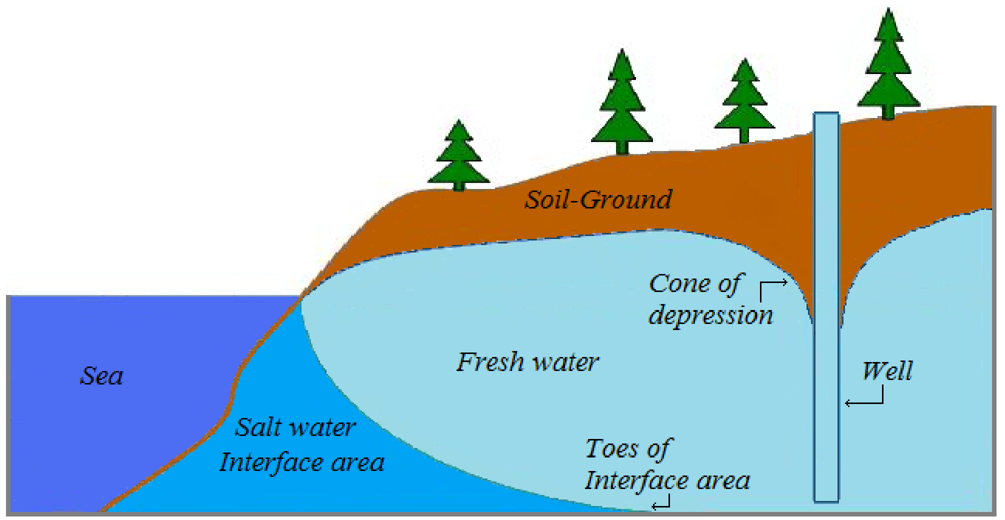
Part 1. Saltwater intrusion

Basic description of the problem.



Saltwater intrusion in coastal aquifer

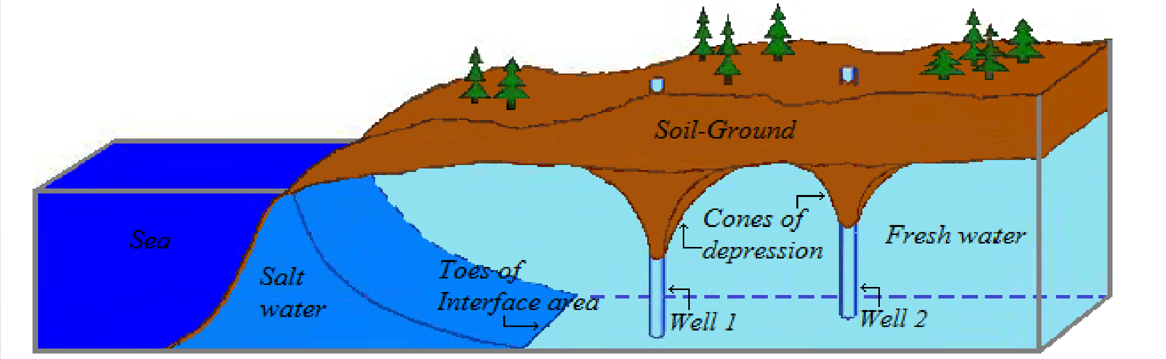
The problem:

Rapidly ***increasing needs for fresh water*** in coastal areas and islands, due to:

- Population growth
- Tourism
- Agriculture needs

Actions:

- ***Intensive pumping*** in freshwater aquifers, mostly during summer months, beyond the tolerable limits of their natural replenishment.



Saltwater intrusion in coastal aquifer

Results:

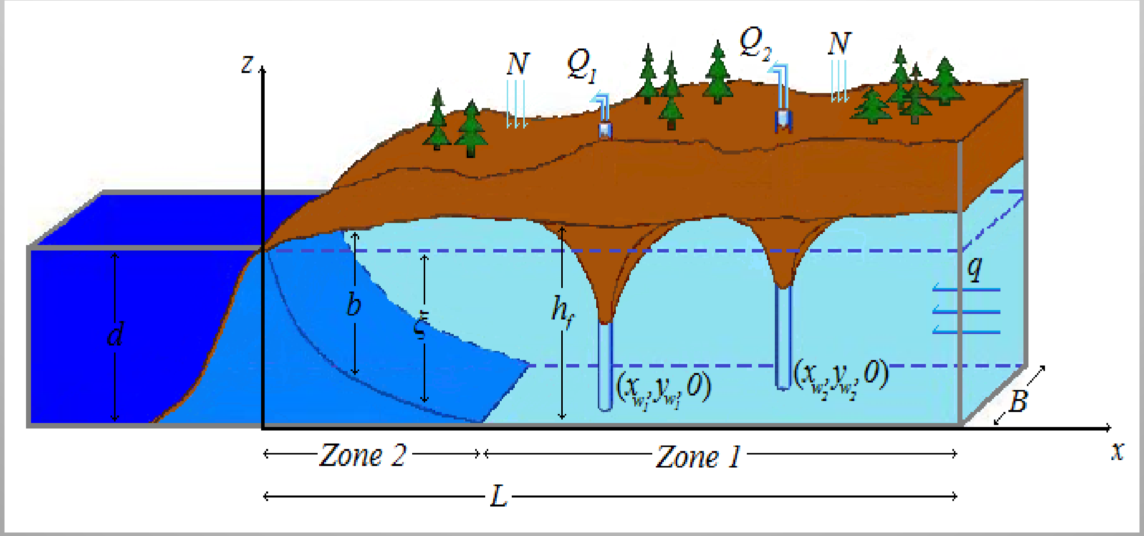
- *Decrease in freshwater table level* in these areas.
- ***Sea water intrusion*** into the coastal aquifer.
- Mixing of fresh and salt water creates *water quality problems*.
- Wells of the area becoming unusable for water supply and irritation.
- Negative ***economical impacts*** in these areas.

There is a great need for developing ***pumping management methodologies***, in order to determine:

- the ***total volume of water*** that can be pumped from coastal aquifers, while protecting the wells from saltwater intrusion.
- the ***optimal places*** where wells can be placed, in order to maximize the non-risk pumping of fresh water.
- the ***max number of wells*** that can be distributed over the aquifer.

Part 2. Mathematical approach-model equations

Presentation of the ***mathematical model simplifications*** and equations, we use to describe the water flow inside the aquifer.



Model simplifications

Flow in coastal aquifers is a very complex process, because

- there exist more than one *mixing fluid phases*,
- fluid density depends on the unknown concentrations,
- there exist a great spatial *variability of hydraulic parameters* inside the region of the aquifer.

So, model simplifications are needed, in order to provide reasonable approximate predictions. The most common of them are:

- ***Sharp Interface*** approximation,
- ***Ghyben-Herzberg*** equation.

Sharp Interface approximation:

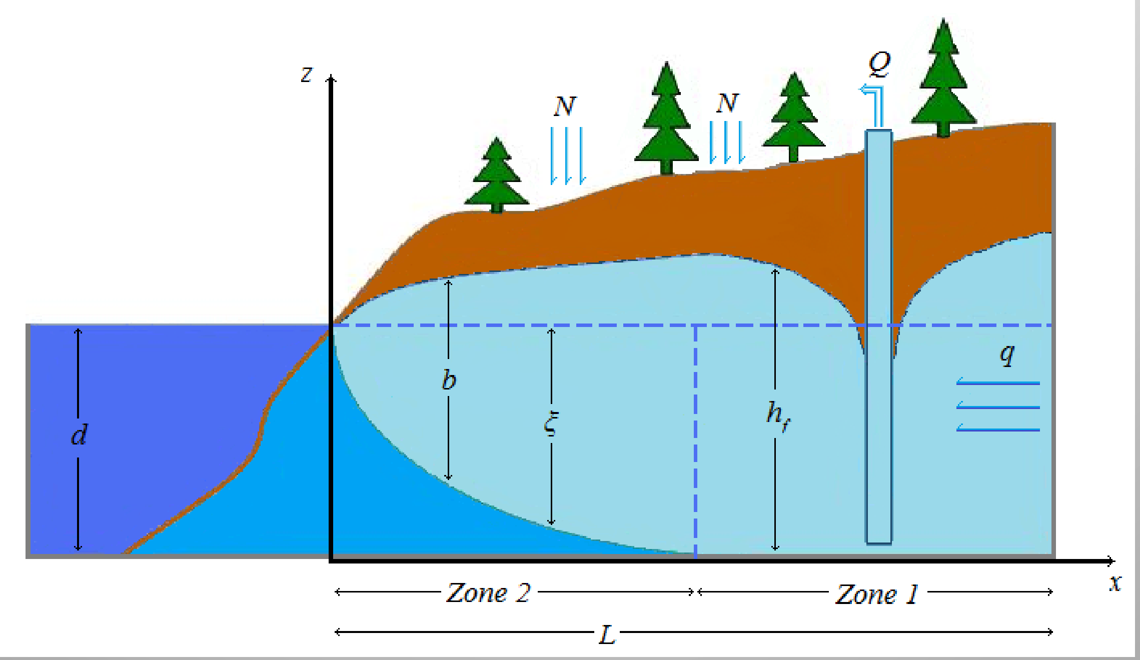
- There is no direct mixing of sea and fresh water inside the aquifer.
- There exists an ***Interface area***, the movement and position of which we are trying to study.

Ghyben-Herzberg equation:

$h_f - d = \delta\xi$, where $\delta = \dfrac{\rho_{salt} - \rho_{fresh}}{\rho_{fresh}} \cong 0.025.$

- We can use this equation only if the Interface area is practically stabilized at some position, i.e. when the flow conditions approach ***steady state***.

Aquifer 2D: Mathematical parameters



Parameters of the aquifer (distances)

- L: Length of the aquifer.
- B: Width of the aquifer.
- d: Height of the aquifer from its bottom to sea level.
- $b(x,y)$: Freshwater depth from free surface to the Interface.
- $\xi(x,y)$: Freshwater depth from the sea level to the Interface.
- $h_f(x,y)$: Freshwater piezometric head with reference to the bottom of the aquifer.

Parameters of the aquifer (areas and water movement)

- τ: Points where the interface surface intersects the base of the aquifer (***Toes area***).
- ***Q (m³/day)***: ***Pumping rates*** of the aquifer wells.
- *N (mm/year)*: Water recharge distributed over the surface of the aquifer (e.g. rain, rivers).
- *K (m/day)*: Hydraulic conductivity.
- *q (m²/day)*: Ambient horizontal discharge per unit width of the aquifer.

Equations:

- <u>Zone 1</u>. ***Steady flow*** equation:

$$\frac{\partial}{\partial x}\left(Kh_f\frac{\partial h_f}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh_f\frac{\partial h_f}{\partial y}\right)+N-Q=0$$

where $h_f = b$.

- <u>Zone 2</u>. ***Steady flow*** equation:

$$\frac{\partial}{\partial x}\left(Kb\frac{\partial h_f}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kb\frac{\partial h_f}{\partial y}\right)+N-Q=0$$

where $h_f = b+d-\xi$.

Following *Strack*[1976], we define the ***flow potential*** $\varphi=\varphi(x,y)$ as follows:

- <u>Zone 1</u>.

$$\varphi = \frac{1}{2}\left(h_f^2 - (1+\delta)d^2\right)$$

- <u>Zone 2</u>.

$$\varphi = \frac{(1+\delta)}{2\delta}\left(h_f - d\right)^2$$

- <u>Toes of interface area</u>.

$\xi = \delta$, $h_f = (1+\delta)d$ and $\varphi_T = \dfrac{(1+\delta)\delta}{2} d^2$

The ***flow potential*** $\varphi=\varphi(x,y)$ is a continuous and smooth function across the boundary between zones 1 and 2 and satisfies the differential equation:

$$\frac{\partial}{\partial x}\left(K\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial\varphi}{\partial y}\right)+N-Q=0$$

with *boundary conditions*:

coast boundary (x=0): $\xi=0$, $\varphi(0,y)=0$

$$q_n=-Kb\frac{\partial h_f}{\partial n}=0\Rightarrow\frac{\partial h_f}{\partial n}=0$$

(i.e. at no flow boundaries the flow towards direction n perpendicular to the boundary is 0).

If *K*, *N*, *Q* and the boundary conditions are known, the previous equation can be solved for $\varphi(x,y)$ using analytical or numerical methods. Once $\varphi(x,y)$ is determined, the interface surface can be calculated as a function of φ, as follows:

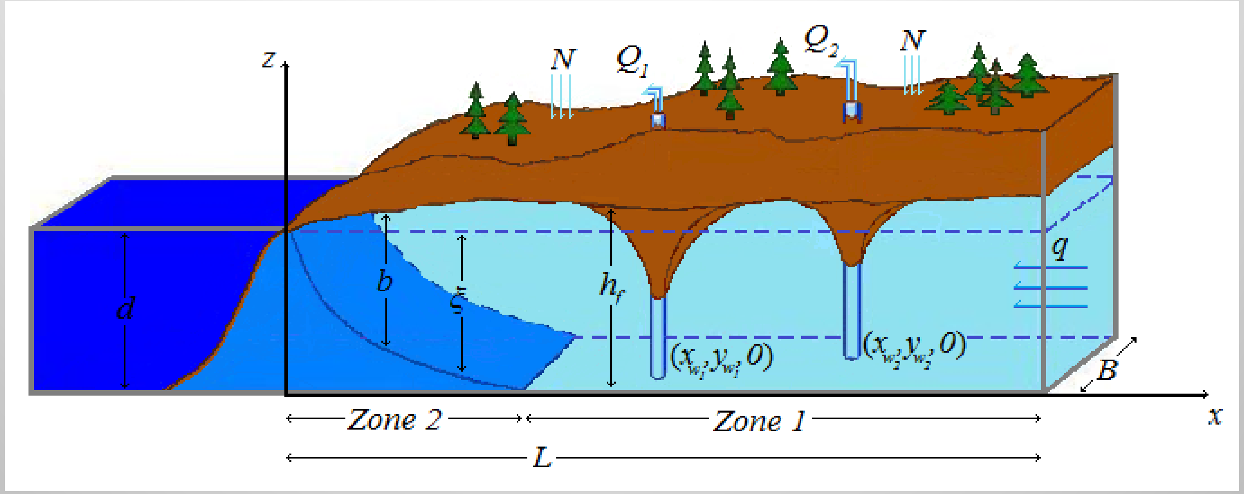
- Zone 1.

$$z = 0,\ h_f = \sqrt{2\varphi + (1+\delta)d^2}, \text{ for } \frac{(1+\delta)\delta}{2}d^2 \le \varphi$$

- Zone 2.

$$z = d - \xi$$

$$\xi = \sqrt{\frac{2\varphi}{\delta(1+\delta)}},\ h_f = \sqrt{\frac{2\delta\varphi}{1+\delta}} + d, \text{ for } 0 \le \varphi \le \frac{(1+\delta)\delta}{2}d^2$$

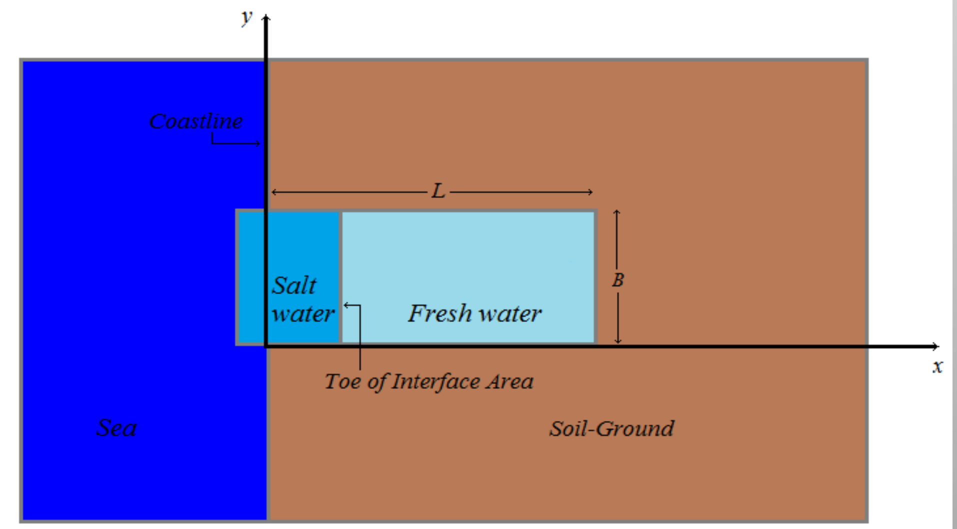


Finally, the ***locus of the Toes of Interface area*** can be determined by solving for x_T, as a function of y_T, the following nonlinear equation:

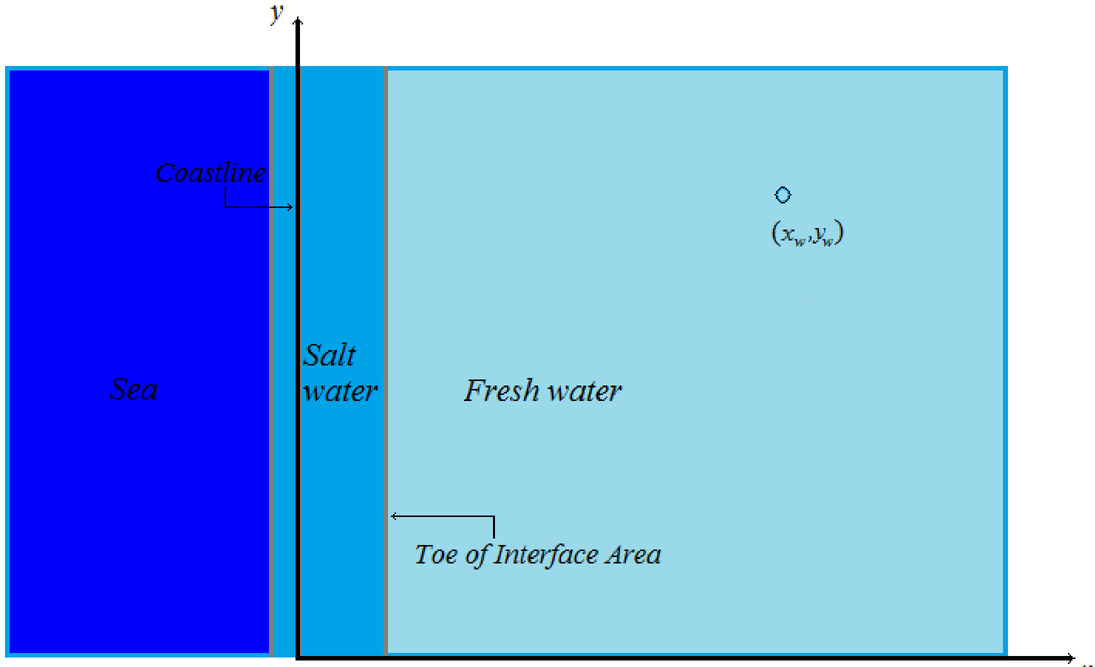
$$\varphi(x_T, y_T) = \frac{(1+\delta)\delta}{2} d^2$$

<u>Part 3</u>. Types of coastal freshwater aquifers

Presentation of three types of coastal freshwater aquifers, with different sets of boundaries. Analytical solution of ***flow potential*** $\varphi=\varphi(x,y)$ for these aquifers.



Aquifer Type C: Impervius up, down and right boundaries



Aquifer Type A: Semi-infinite dimensions

Aquifer type A. Semi infinite dimensions.

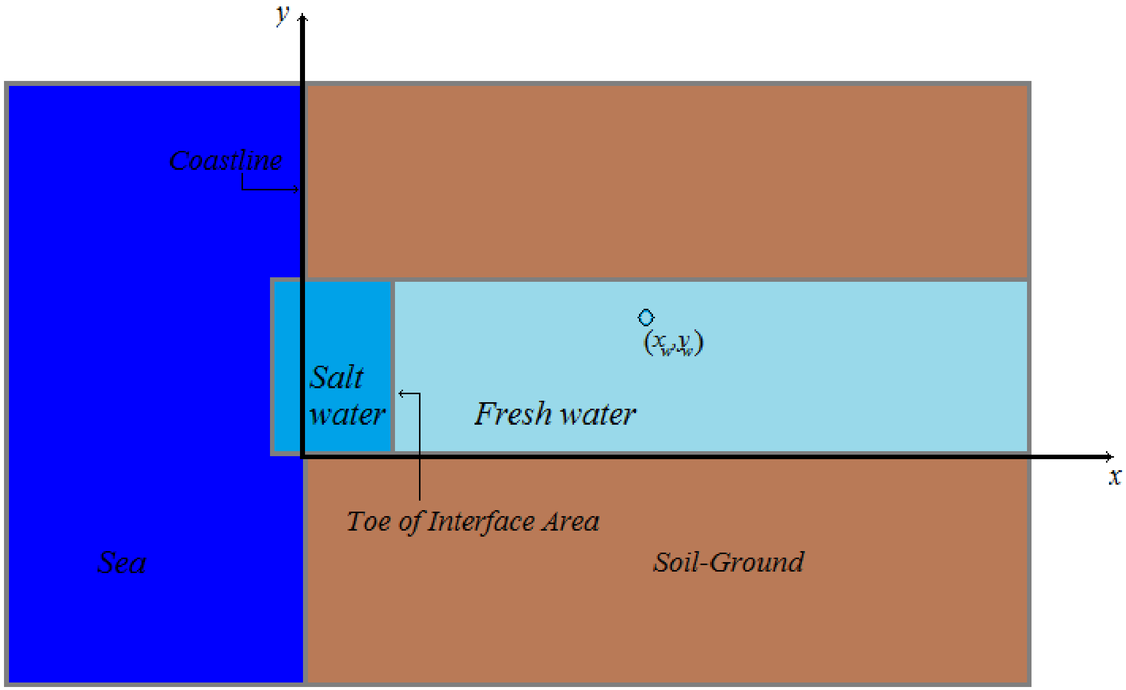
Homogeneous aquifer bounded only on one side by the coastline.

- Analytical solution of ***flow potential***

  (*Strack*[1976], *Cheng*[2000]):

$$\varphi(x,y) = \frac{qx}{K} + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{(x-x_j)^2+(y-y_j)^2}{(x+x_j)^2+(y-y_j)^2}\right]$$

where $(x_j, y_j), j=1,..,M$ are the coordinates of the wells.



Aquifer Type B: Impervius up and down boundaries

Aquifer type B. Infinite length.

Homogeneous aquifer bounded on the left side by the coastline, with up and down impervious boundaries.

- Analytical solution of ***flow potential:***

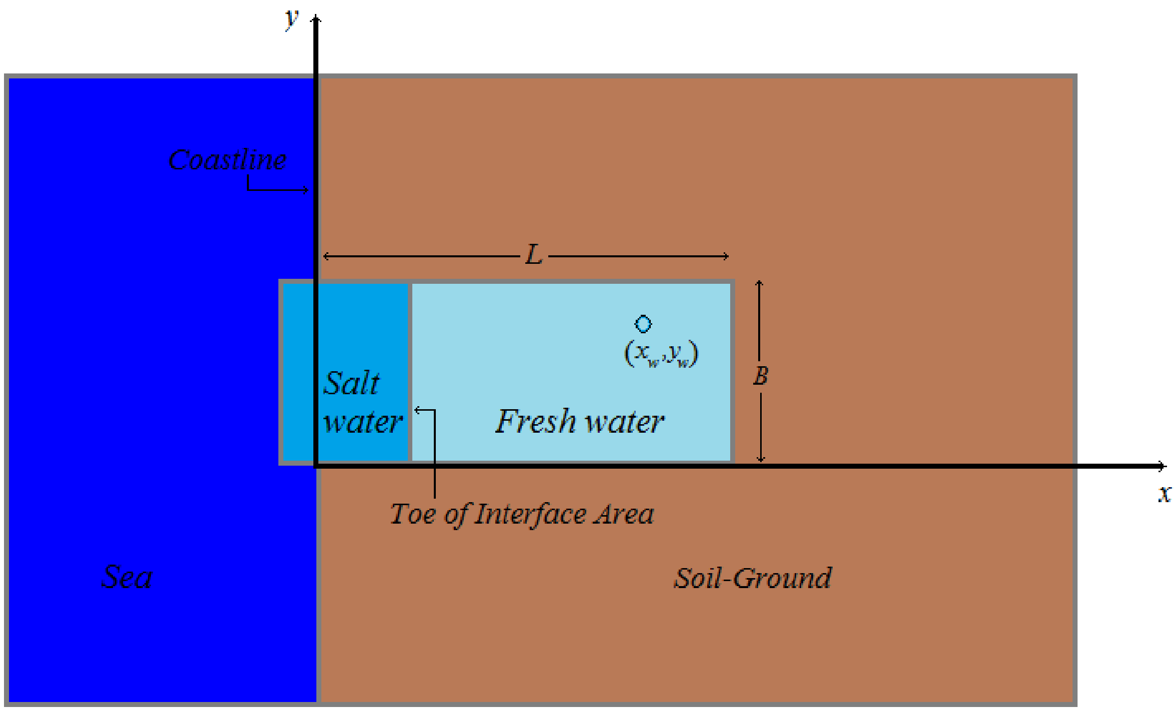
$$\varphi(x,y) = \frac{qx}{K} +$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+(y-y_j)^2}{(x+x_j)^2+(y-y_j)^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+(y-y_j)^2}{(x+x_j)^2+(y+y_j)^2}\right]$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+(y-(2B-y_j))^2}{(x+x_j)^2+(y-(2B-y_j))^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+(y-(2B+y_j))^2}{(x+x_j)^2+(y-(2B+y_j))^2}\right]$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+(y-(-2B+y_j))^2}{(x+x_j)^2+(y-(-2B+y_j))^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+(y-(-2B-y_j))^2}{(x+x_j)^2+(y-(-2B-y_j))^2}\right]$$

where (x_j, y_j), $j=1,..,M$ are the coordinates of the wells.



Aquifer Type C : Impervius up, down and right boundaries

Aquifer type C. Rectangular shape.

Homogeneous aquifer bounded on the left side by the coastline with up, down and right impervious boundaries.

- Analytical solution of ***flow potential***:

$$
\begin{aligned}
\varphi(x,y) = {} & \frac{qx}{K} + \frac{Nx(L-\frac{x}{2})}{K} + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{(x-x_j)^2+(y-y_j)^2}{(x+x_j)^2+(y-y_j)^2}\right] \\
& + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{[x-(2L-x_j)]^2+\left(y-y_j\right)^2}{[x-(2L+x_j)]^2+\left(y-y_j\right)^2}\right] + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{[x+(2L+x_j)]^2+(y-y_j)^2}{[x+(2L-x_j)]^2+(y-y_j)^2}\right] \\
& + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{(x-x_j)^2+(y+y_j)^2}{(x+x_j)^2+(y+y_j)^2}\right] + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{[x-(2L-x_j)]^2+(y+y_j)^2}{[x-(2L+x_j)]^2+(y+y_j)^2}\right] \\
& + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{[x+(2L+x_j)]^2+(y+y_j)^2}{[x+(2L-x_j)]^2+(y+y_j)^2}\right] + \sum_{n=1}^{2} \left\{ \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{(x-x_j)^2+[y-\left(2nB-y_j\right)]^2}{(x+x_j)^2+[y-\left(2nB-y_j\right)]^2}\right] \right. \\
& + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{[x-\left(2L-x_j\right)]^2+[y-\left(2nB-y_j\right)]^2}{[x-\left(2L+x_j\right)]^2+[y-\left(2nB-y_j\right)]^2}\right] + \sum_{j=1}^{M} \frac{Q_j}{4\pi K}\, ln\left[\frac{[x+\left(2L+x_j\right)]^2+[y-\left(2nB-y_j\right)]^2}{[x+\left(2L-x_j\right)]^2+[y-\left(2nB-y_j\right)]^2}\right]
\end{aligned}
$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+[y-(2nB+y_j)]^2}{(x+x_j)^2+[y-(2nB+y_j)]^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{[x-(2L-x_j)]^2+[y-(2nB+y_j)]^2}{[x-(2L+x_j)]^2+[y-(2nB+y_j)]^2}\right]$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{[x+(2L+x_j)]^2+[y-(2nB+y_j)]^2}{[x+(2L-x_j)]^2+[y-(2nB+y_j)]^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+[y-(-2nB+y_j)]^2}{(x+x_j)^2+[y-(-2nB+y_j)]^2}\right]$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{[x-(2L-x_j)]^2+[y-(-2nB+y_j)]^2}{[x-(2L+x_j)]^2+[y-(-2nB+y_j)]^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{[x+(2L+x_j)]^2+[y-(-2nB+y_j)]^2}{[x+(2L-x_j)]^2+[y-(-2nB+y_j)]^2}\right]$$

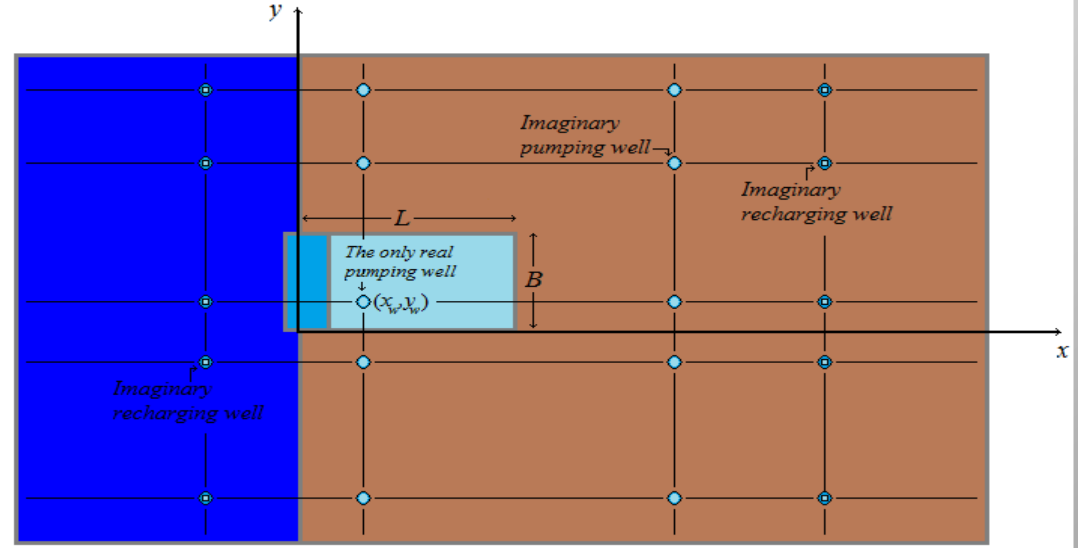
$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{(x-x_j)^2+[y-(-2nB-y_j)]^2}{(x+x_j)^2+[y-(-2nB-y_j)]^2}\right]+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{[x-(2L-x_j)]^2+[y-(-2nB-y_j)]^2}{[x-(2L+x_j)]^2+[y-(-2nB-y_j)]^2}\right]$$

$$+\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\,ln\left[\frac{[x+(2L+x_j)]^2+[y-(-2nB-y_j)]^2}{[x+(2L-x_j)]^2+[y-(-2nB-y_j)]^2}\right]\Bigg\}$$

where (x_j, y_j), $j=1,..,M$ are the coordinates of the wells.

Part 4. Pumping optimization methods in coastal aquifers

Our goal is to achieve the maximum pumping rates of all the wells inside the aquifer, without risking the saltwater contamination of the wells, known as the ***Toe Constraint*** formulation.



Method of Images: One real pumping and many imaginary pumping and recharging wells

***AL.O.P.EX.** method (**AL**gorithm **O**f **P**attern **EX**traction)*

- Introduced by *Harth* και *Tzanakou*, Syracuse University, 1974.
- *Stochastic optimization for adaptive correction of atmospheric distortion in astronomical observation, M. Zakynthinaki*, PhD Thesis, Chania, 2001.

- ***Stochastic optimization algorithm***.
- Control (cost-profit) function $f=f(x_1,x_2,x_3,x_4,..,x_n)$.
- Goal: Maximize or minimize the control function.
- Local extrema can be avoided by the use of some kind of noise.

A few words about the ***ALOPEX*** method

- *Iterative* algorithm.
- Every iteration starts with the data of the previous one.
- In every iteration all the control variables of the cost function can be changed simultaneously.
- The new values of the variables are ***stochastically*** depended from the change of the cost function between two iterations.
- The *stochastic element* of the procedure is the ***noise***, which is controlled from the user.

Versions of the **ALOPEX** algorithm

- ***ALOPEX I***

$$x^{(n)} = x^{(n-1)} + \delta^{(n)}$$

where: $\delta^{(n)} := \left(\delta_1^{(n)}, \delta_2^{(n)}, \dots, \delta_N^{(n)}\right)$, $\delta_i^{(n)} = \begin{cases} \delta, & \text{with probability} \quad p_i^{(n)} \\ -\delta, & \text{with probability} \quad 1 - p_i^{(n)} \end{cases}$.

- ***ALOPEX II***

$$x^{(n)} = x^{(n-1)} + c\Delta x^{(n)} \Delta f^{(n)} + g^{(n)}$$

where c:constant.

- ***ALOPEX IIIa***

$$x^{(n)} = x^{(n-1)} + c^{(n)} \Delta x^{(n)} \Delta f^{(n)} + g^{(n)}$$

where: $c^{(n)} = -\dfrac{{g^{(n)}}^T \Delta x^{(n)}}{{\Delta x^{(n)}}^T H \Delta x^{(n)} \Delta f^{(n)}}$.

Also: $x^{(n)} := \left(x_1^{(n)}, x_2^{(n)}, \dots, x_N^{(n)}\right)$ $\quad g^{(n)} := \left(g_1^{(n)}, g_2^{(n)}, \dots, g_N^{(n)}\right)$

$\Delta x^{(n)} := x^{(n-1)} - x^{(n-2)}$ $\quad \Delta f^{(n)} := f\left(x^{(n-1)}\right) - f(x^{(n-2)})$

- ***ALOPEX III***

$$x^{(n)} = x^{(n-1)} + c\Delta x^{(n)}\Delta f^{(n)} + g^{(n)}$$

where $c^{(n)}$: interpolation approximation of second degree.

- ***ALOPEX IVa***

$$x^{(n)} = x^{(n-1)} + c^{(n)}\Delta x^{(n)} \frac{\Delta f^{(n)}}{|\Delta f^{(n-1)}|} + g^{(n)}$$

where $c^{(n)} = -\frac{{g^{(n)}}^T \Delta x^{(n)}}{{\Delta x^{(n)}}^T H \Delta x^{(n)} \Delta f^{(n)}} \left|\Delta f^{(n-1)}\right|$.

- ***ALOPEX IV***

$$x^{(n)} = x^{(n-1)} + c\Delta x^{(n)} \frac{\Delta f^{(n)}}{|\Delta f^{(n-1)}|} + g^{(n)}$$

where c:constant.

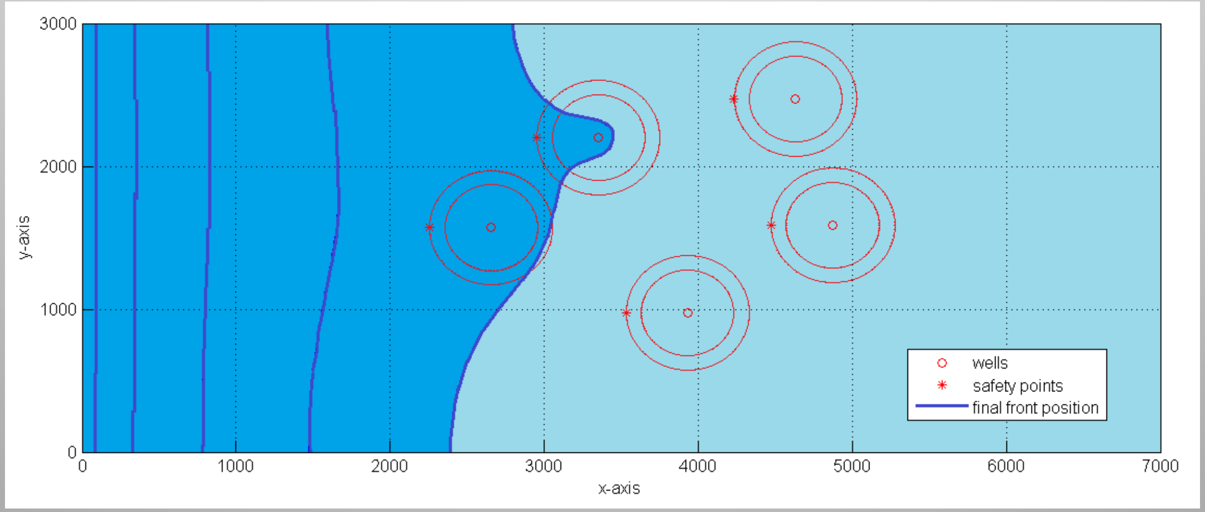
Also:

$$x^{(n)} := \left(x_1^{(n)}, x_2^{(n)}, \dots, x_N^{(n)}\right) \quad g^{(n)} := \left(g_1^{(n)}, g_2^{(n)}, \dots, g_N^{(n)}\right)$$

$$\Delta x^{(n)} := x^{(n-1)} - x^{(n-2)} \quad \Delta f^{(n)} := f\left(x^{(n-1)}\right) - f(x^{(n-2)})$$

Part 5. Numerical simulations

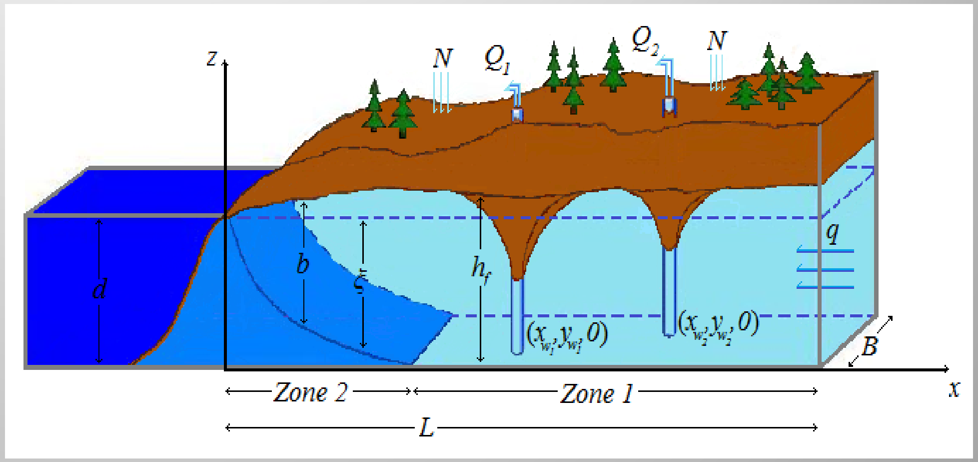
Applications of ***ALOPEX*** algorithm in an Type C aquifer with 2 and 5 wells, using the ***MATLAB*** environment.



Aquifer type C: ***A hypothetical test case***
2 pumping wells

Aquifer's parameters:

- $L=7000\ m$
- $B=3000\ m$
- $(x_{w1},y_{w1})=(1500,700)\ m$
- $(x_{w2},y_{w2})=(2350,2200)\ m$
- $K=100\ m/day$
- $N=30\ mm/year$
- $q=1.23\ m^2\ /day$
- $d=25\ m$
- $Q_{total}=20000\ m^3/day$
- $Q_{local\_min}=(200,200)\ m^3/day$
- $Q_{local\_max}=(2500,2500)\ m^3/day$



Well's cone of depression:

- *radius of influence=300 m*
- *safety distance=100 m.*

ALOPEX II algorithm:

$$Q(i)^{(k)}=Q(i)^{(k-1)}+c*[Q(i)^{(k-1)}-Q(i)^{(k-2)}]*[Profit^{(k-1)}-Profit^{(k-2)}]+noise^{(k)}$$

and Profit function:

$$Profit^{(k)} = e^{-\left(\frac{\sum_{i=1}^{n} Q_{local\_max}(i) - \sum_{i=1}^{n} Q(i)^{(k)}}{\sum_{i=1}^{n} Q_{local\_max}(i)}\right)^2}$$ at k-th iteration.

Algorithm parameters:

- $c=0.6$: acceleration factor
- $noise(i)^{(k)}=0.05*Q(i)^{(k-1)}*(-0.5+1.5*rand)$.

Penalties management:

- Q_{local_min} penalty=*1.20*
- Q_{local_max} penalty=*0.95*
- *x-movement* penalty=*0.95*
- *critical-distance* penalty=*0.95*
- Q_{total} penalty=*1.20.*

Penalties definitions

- ***Q_{local_min} penalty***$\in[1,2]$

for i=1:n
  if $Q(i)^{(k)} \leq Q_{local\_min}(i)$
    $Q(i)^{(k)}=Q_{local\_min}\ penalty * Q(i)^{(k)}$
  end
end, at k-th iteration.

- ***Q_{local_max} penalty*** $\in[0,1]$

for i=1:n
  if $Q(i)^{(k)} \geq Q_{local\_max}(i)$
    $Q(i)^{(k)}=Q_{local\_max}\ penalty * Q(i)^{(k)}$
  end
end, at k-th iteration.

- ***x-movement penalty*** $\in[0,1]$

for i=1:n
  if $x_T(i)^{(k)} \geq safety\text{-}point(i)$
    $Q(i)^{(k)}=x\text{-}movement\ penalty * Q(i)^{(k)}$
  end
end, at k-th iteration.

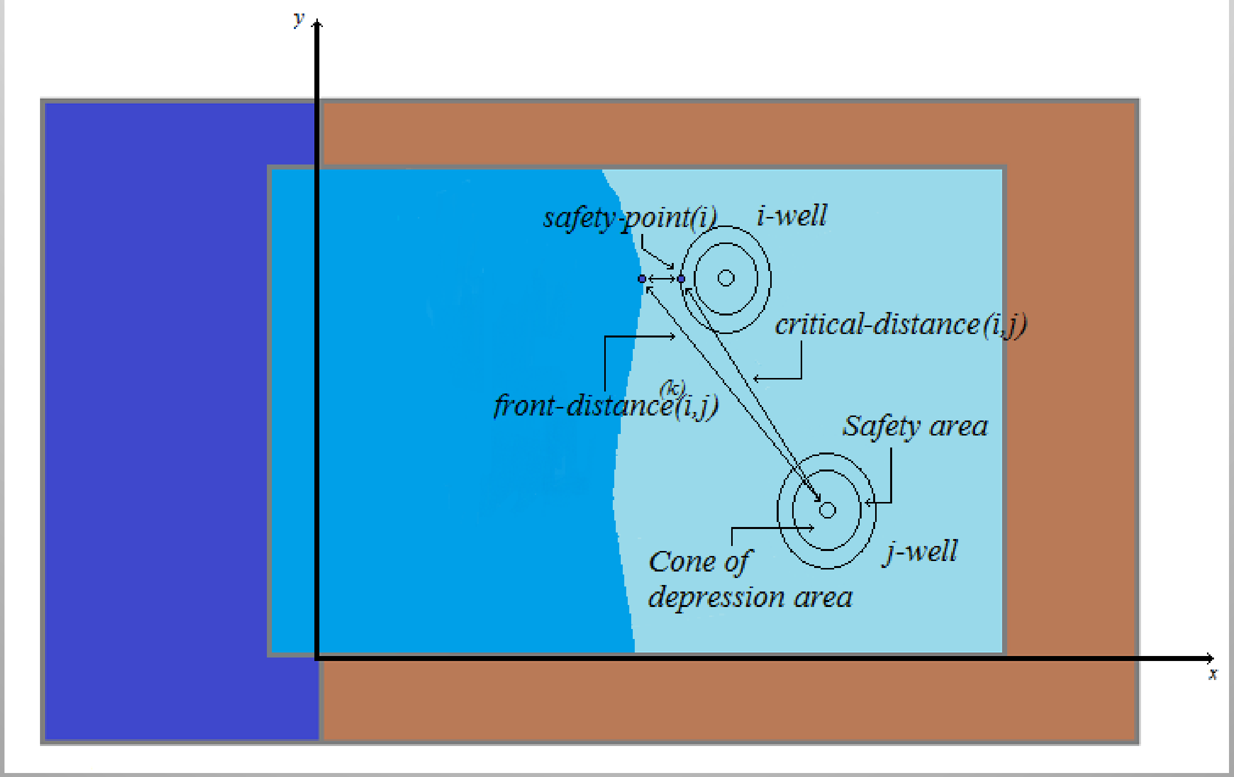
- ***Q_{total} penalty***$\in[1,2]$

$S=\Sigma Q(i)^{(k)}$
for *i=1:n*
    $S1=S- Q(i)^{(k-1)}+ Q(i)^{(k)}$
    if $S1>Q_{total}$
      $Q(i)^{(k)} = Q(i)^{(k)} – Q_{total\_penalty} * (S1-Q_{total})$
    end
    S=S1;
end, at k-th iteration.

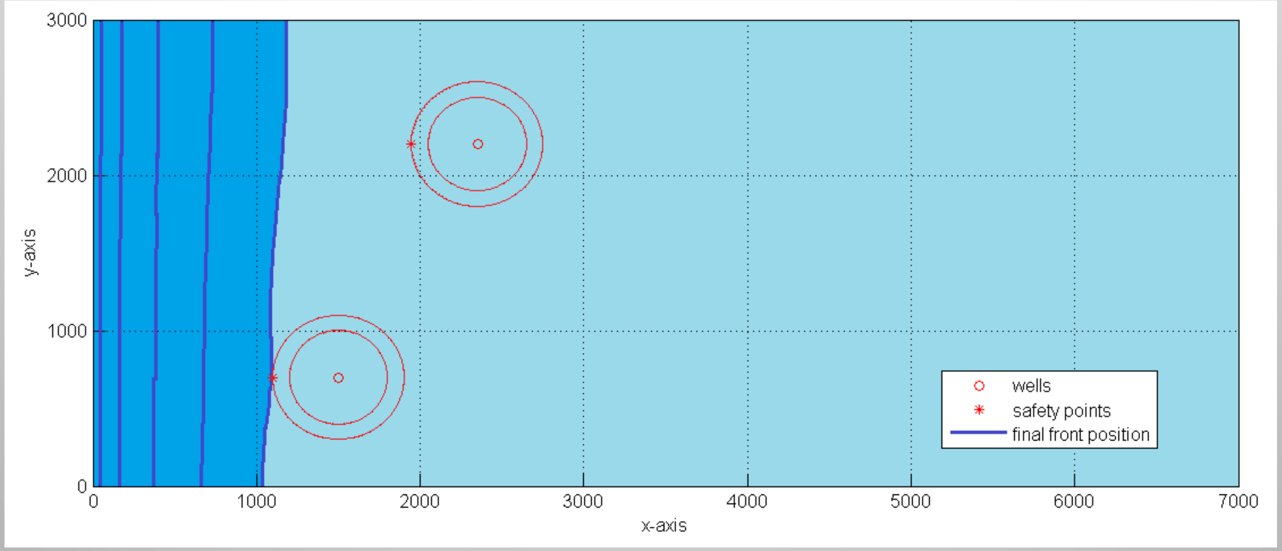
- ***critical-distance penalty*** $\in[0,1]$

for i=1:n
 for j=1:n
  if $front\text{-}distance(i,j)^{(k)} \leq critical\text{-}distance(i,j)$
    $Q(j)^{(k)}=critical\text{-}distance\ penalty * Q(j)^{(k)}$
  end
 end
end,   at k-th iteration, with n:number of wells.

Definition of ***safety points*** and ***cone of depression areas*** for aquifer wells.



Numerical results using the MATLAB environment

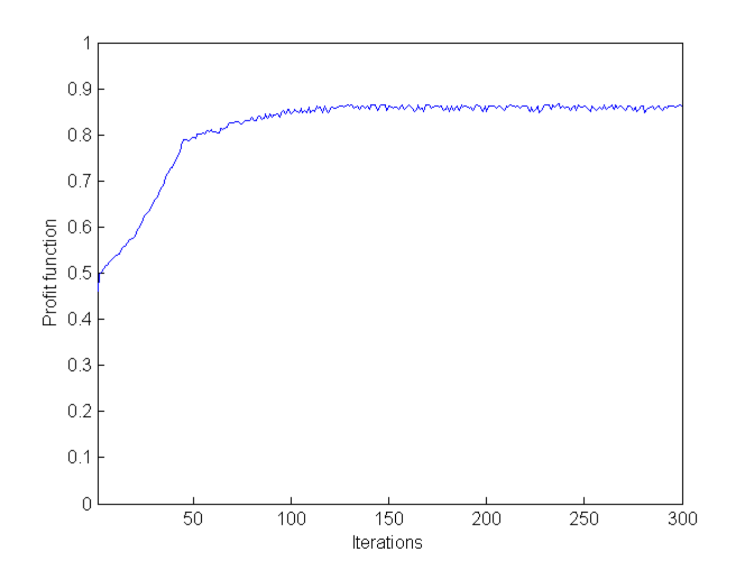


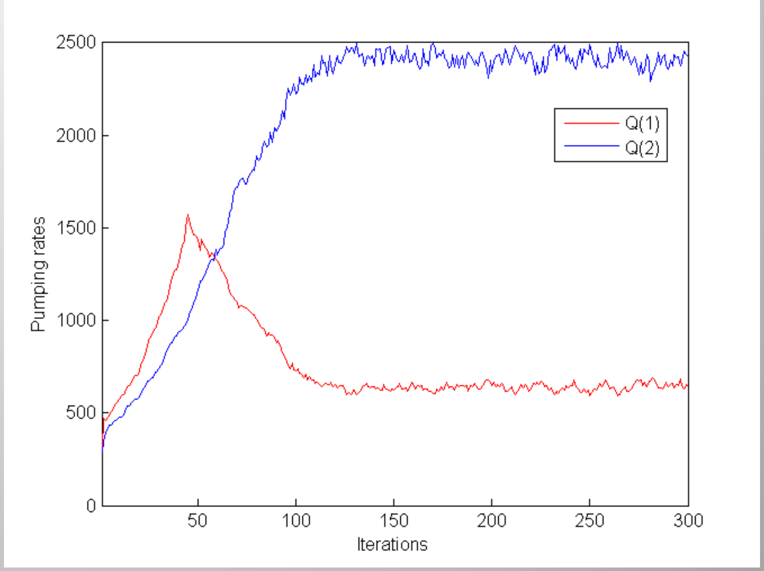
Optimal values for pumping rates:

- $Q^{opt}(1)=606.90\ m^3/day$
- $Q^{opt}(2)=2598.45\ m^3/day$

$$\sum Q^{opt}(i)=3205.35\ m^3/day.$$

Profit function and pumping rates during a typical optimization run of 300 iterations.





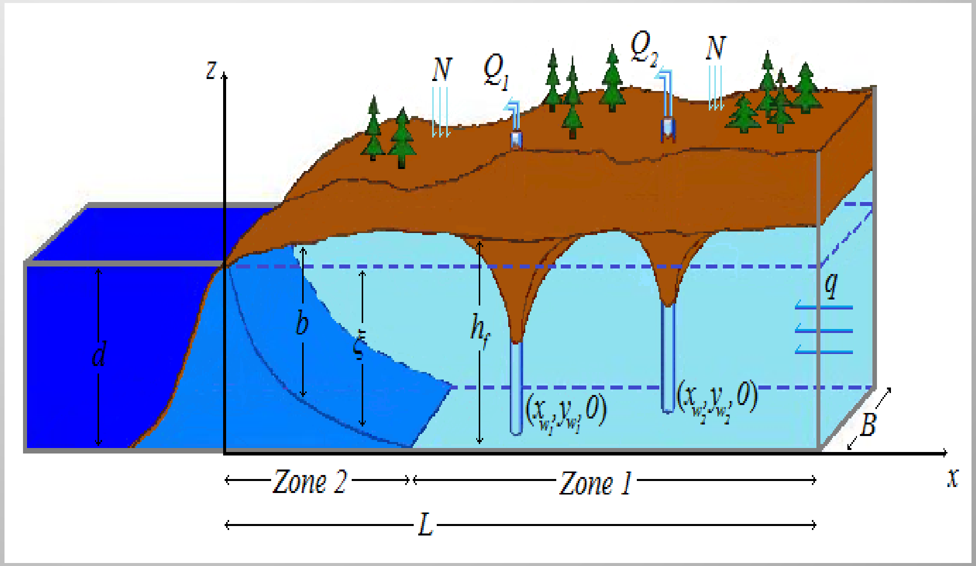
Penalties activation:

- Q_{local_min} penalty: 0 times
- Q_{local_max} penalty: 69 times
- *x-movement* penalty: 139 times
- *critical-distance* penalty: 42 times
- Q_{total} penalty: 0 times.

Aquifer type C: ***Vathi area of Greek island Kalymnos***
5 pumping wells, **Case 1**

Aquifer's parameters:

- *L=7000 m*
- *B=3000 m*
- *(x_{w1},y_{w1})=(3932,975) m*
- *(x_{w2},y_{w2})=(2657,1572) m*
- *(x_{w3},y_{w3})=(4873,1586) m*
- *(x_{w4},y_{w4})=(3353,2200) m*
- *(x_{w5},y_{w5})=(4632,2470) m*
- *K=100 m/day*
- *N=30 mm/year*
- *q=1.23 m^2 /day*
- *d=25 m*



- *Q_{total}=20000 m^3/day*
- *$Q_{local\_min}(i)$=200 m^3/day*
- *$Q_{local\_max}(i)$=1500 m^3/day*

Well's cone of depression:

- *radius of influence=300 m*
- *safety distance=100 m.*

Case 1

ALOPEX II algorithm:

$$Q(i)^{(k)}=Q(i)^{(k-1)}+c*[Q(i)^{(k-1)}- Q(i)^{(k-2)}]*[Profit^{(k-1)}- Profit^{(k-2)}]+noise^{(k)}$$

and Profit function:

$$Profit^{(k)} = e^{-\left(\frac{\sum_{i=1}^{n} Q_{local\_max}(i) - \sum_{i=1}^{n} Q(i)^{(k)}}{\sum_{i=1}^{n} Q_{local\_max}(i)}\right)^2}$$ at k-th iteration.

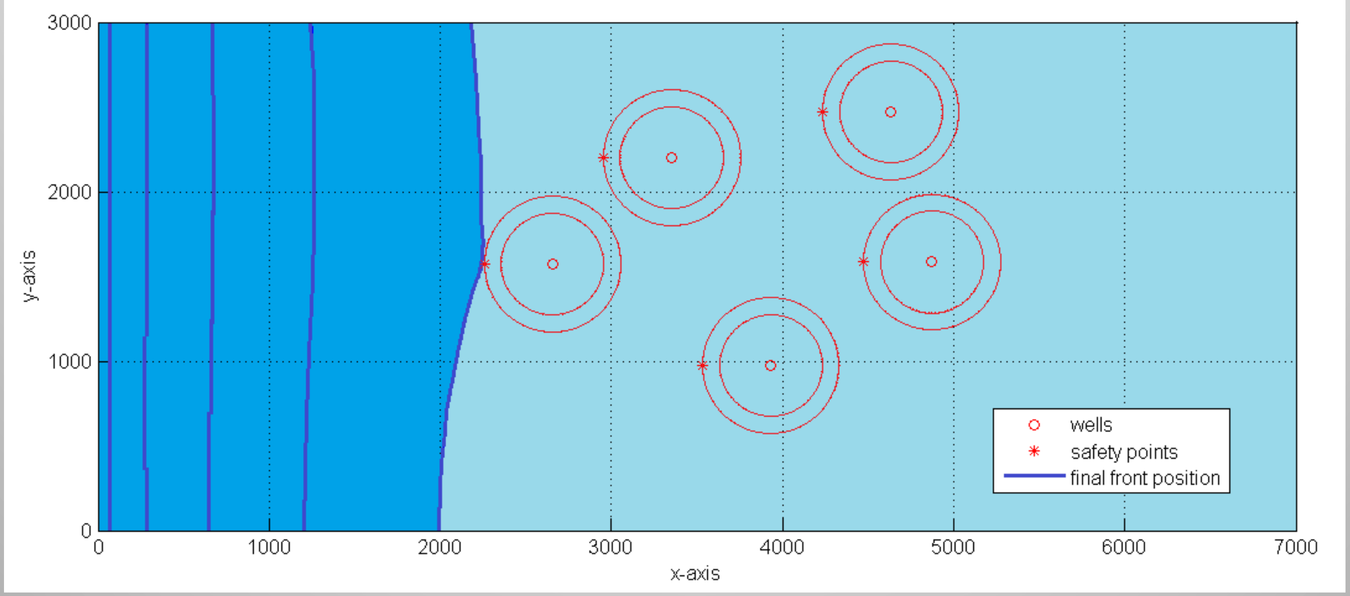
Algorithm parameters:

- $c=0.6$: acceleration factor
- $noise(i)^{(k)}=0.05*Q(i)^{(k-1)}*(-0.5+1.5*rand)$.

Penalties management:

- Q_{local_min} penalty$=1.20$
- Q_{local_max} penalty$=0.95$
- *x-movement* penalty$=0.95$
- *critical-distance* penalty$=0.95$
- Q_{total} penalty$=1.20$.

Case 1: Numerical results using the MATLAB environment



Optimal values for pumping rates:

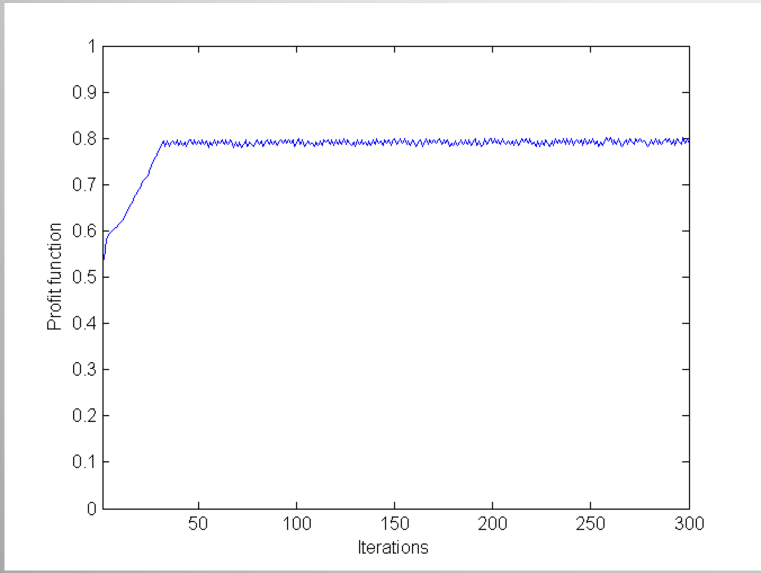
- $Q^{opt}(1)=875.12\ m^3/day$
- $Q^{opt}(2)=237.81\ m^3/day$
- $Q^{opt}(3)=847.42\ m^3/day$
- . $Q^{opt}(4)=596.65\ m^3/day$
- $Q^{opt}(5)=1402.19\ m^3/day$

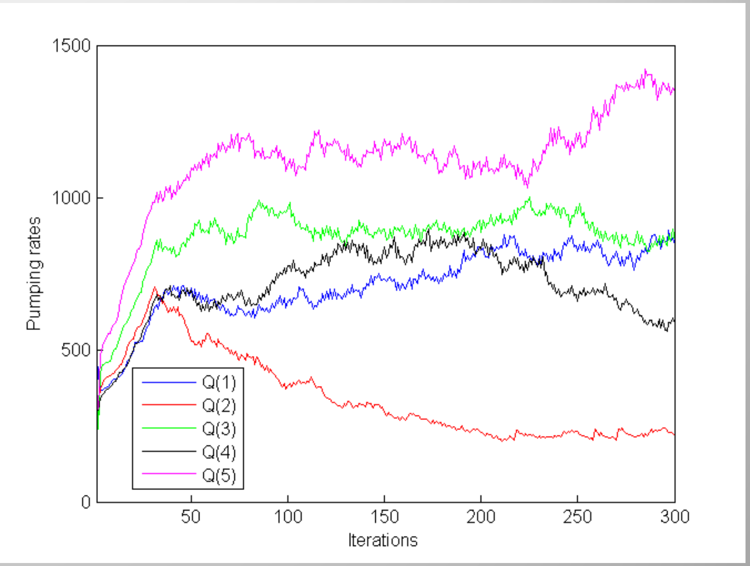
$\sum Q^{opt}(i)=3959.19\ m^3/day.$

Penalties activation:

- Q_{local_min} penalty: 3 times
- Q_{local_max} penalty: 0 times
- *x-movement* penalty: 175 times
- *critical-distance* penalty: 128 times
- Q_{total} penalty: 0 times.

<u>Case 1:</u> ***Profit function*** and ***pumping rates*** during a typical optimization run of 300 iterations.





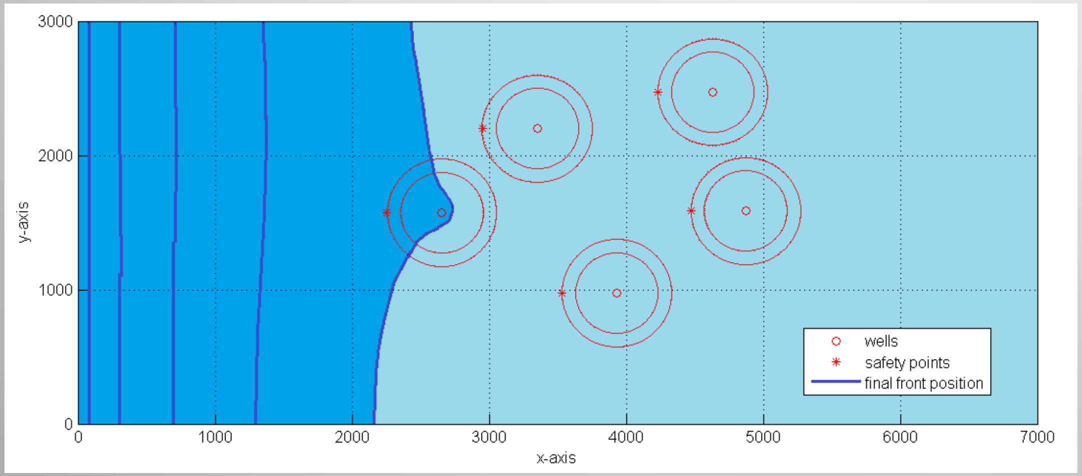
Stopping criterion

difference_of_means(k) =abs(mean(Profit(k-10:k))-mean(Profit(k-20:k-10)))
standard_deviation(k) = std(Profit(k-10:k))

if difference_of_means(k)<0.00005 and standard_deviation(k)<0.01
  conv=true
end

Aquifer type C: ***Vathi area of Greek island Kalymnos***
Sensitivity analysis

Case 1a. *Optimal values for pumping rates increased by a factor of 2%.*



Aquifer properties, well properties and penalties management same as in original case.

Modified pumping rates:

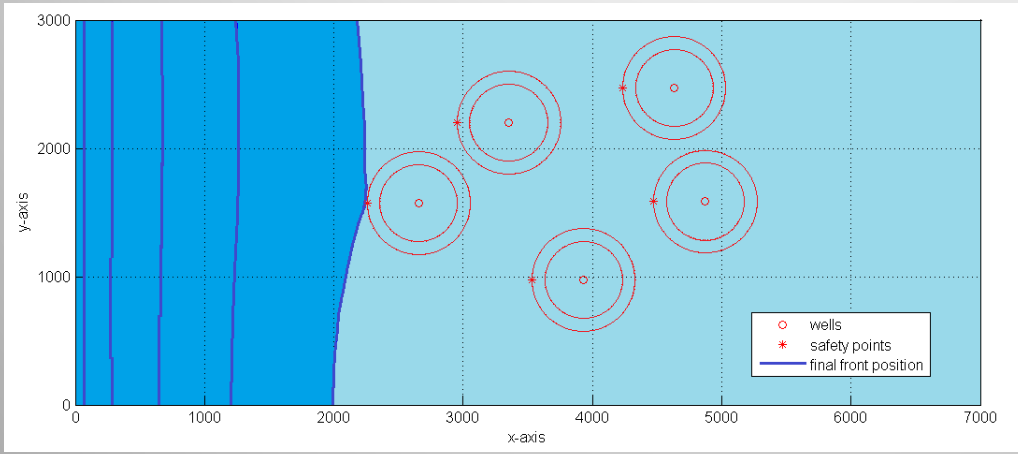
- $Q^{mod}(1)=1.02*875.12 = 892.62\ m^3/day$
- $Q^{mod}(2)=1.02*237.81 = 242.57\ m^3/day$
- $Q^{mod}(3)=1.02*847.42 = 864.37\ m^3/day$
- $Q^{mod}(4)=1.02*596.65 = 608.58\ m^3/day$
- $Q^{mod}(5)=1.02*1402.19 = 1430.23\ m^3/day$

$\sum Q^{mod}(i)=4038.37\ m^3/day.$

Aquifer type C: ***Vathi area of Greek island Kalymnos***

Sensitivity analysis

<u>Case 1b.</u> *Rain factor N increased by a value of 20%.*



Aquifer properties, well properties and penalties management same as in original case.

N^{new}=1.20*30=36 mm/year

Optimal values for pumping rates:

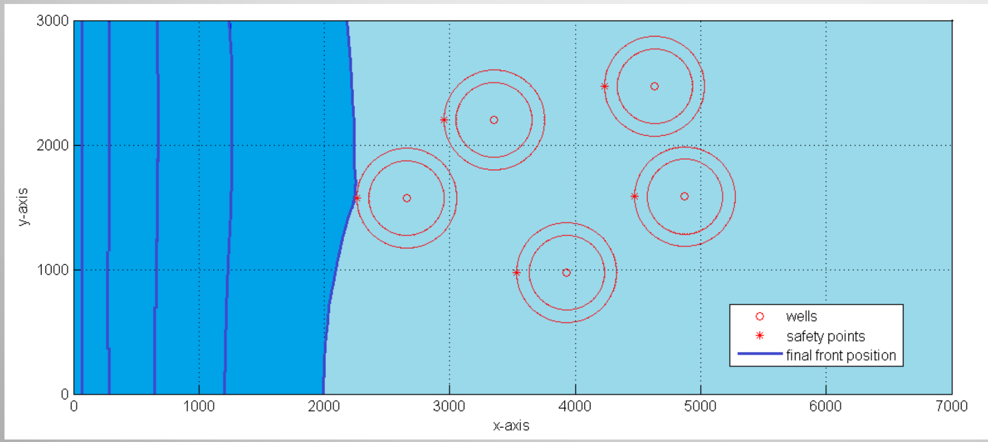
- $Q^{opt}(1) = 920.08\ m^3/day$
- $Q^{opt}(2) = 319.56\ m^3/day$
- $Q^{opt}(3) = 1323.68\ m^3/day$
- $Q^{opt}(4) = 624.65\ m^3/day$
- $Q^{opt}(5) = 1046.31\ m^3/day$

$\sum Q^{opt}(i) = 4234.28\ m^3/day$

(6.95% increase of the sum of pumping rates from original case)

Aquifer type C: ***Vathi area of Greek island Kalymnos***
Sensitivity analysis

<u>Case 1c.</u> *Ambient water discharge factor q decreased by a value of 20%.*



Aquifer properties, well properties and penalties management same as in original case.

$\boldsymbol{q^{new}=0.80*1.23=0.984\ m^2/day}$

Optimal values for pumping rates:

- $Q^{opt}(1)= 863.12\ m^3/day$
- $Q^{opt}(2)= 220.70\ m^3/day$
- $Q^{opt}(3)= 714.73\ m^3/day$
- $Q^{opt}(4)= 586.88\ m^3/day$
- $Q^{opt}(5)= 856.59\ m^3/day$

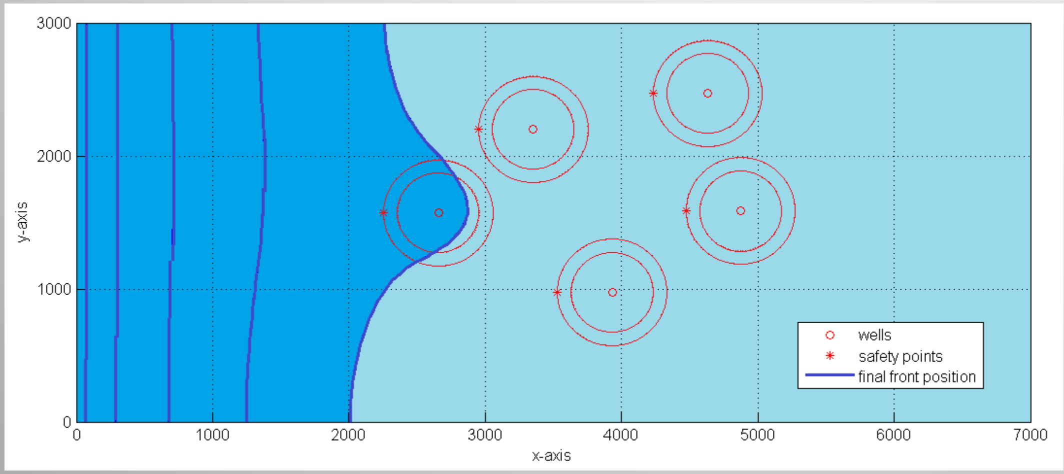
$\sum Q^{opt}(i)= 3242.02\ m^3/day$

(18.1% decrease of the sum of pumping rates from original case)

Aquifer type C: ***Vathi area of Greek island Kalymnos***

Sensitivity analysis

<u>Case 1d.</u> *Turning off the x-movement penalty.*



Aquifer properties and well properties same as in original case.

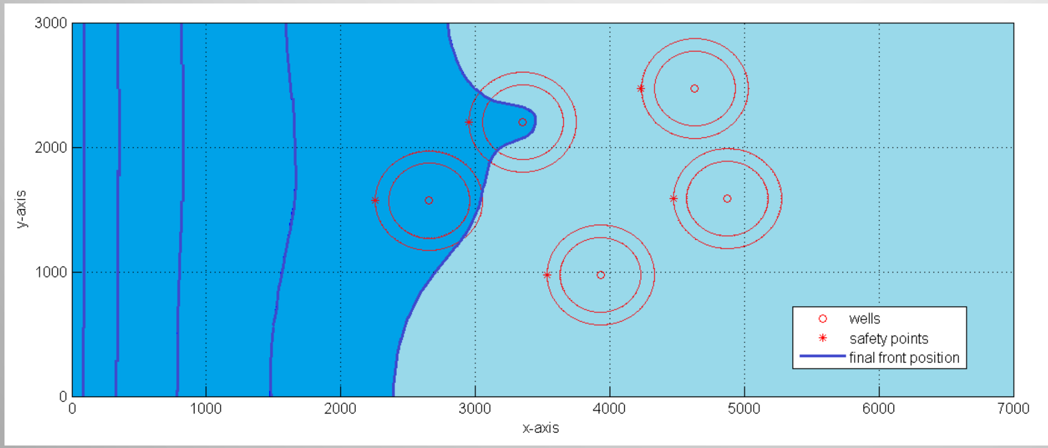
Penalties management:

- Q_{local_min} penalty=*1.20*
- Q_{local_max} penalty=*0.95*
- ***x-movement* penalty=*1.00***
- *critical-distance* penalty=*0.98*

- Results: ***Saltwater intrusion***
- Actions: Decrease the *critical-distance* penalty to the area of 0.95 (lack of fine-tuning in the convergence procedure)

Aquifer type C: ***Vathi area of Greek island Kalymnos***
Sensitivity analysis

Case 1e. *Turning off the critical-distance penalty.*



Aquifer properties and well properties same as in original case.

Penalties management:

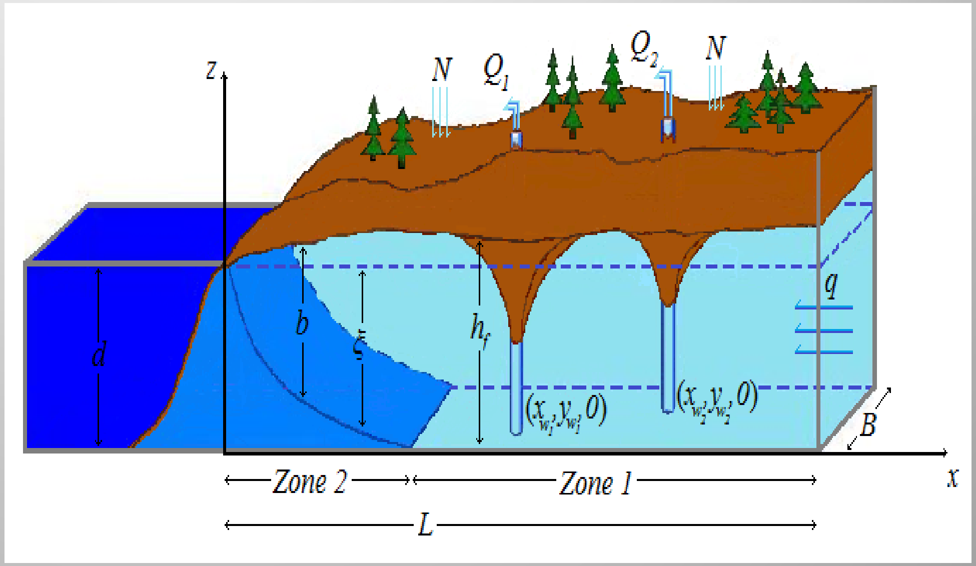
- Q_{local_min} penalty=*1.20*
- Q_{local_max} penalty=*0.95*
- *x-movement* penalty=*0.95*
- ***critical-distance* penalty=*1.00***

- Results: ***Saltwater intrusion***
- Actions: Decrease the *x-movement* penalty to the area of 0.70 (lack of fine-tuning in the convergence procedure)

Aquifer type C: ***Vathi area of Greek island Kalymnos***
5 pumping wells, **<u>Case 2</u>**

Aquifer's parameters:

- $L=7000\ m$
- $B=3000\ m$
- $(x_{w1},y_{w1})=(3932,975)\ m$
- $(x_{w2},y_{w2})=(2657,1572)\ m$
- $(x_{w3},y_{w3})=(4873,1586)\ m$
- $(x_{w4},y_{w4})=(3353,2200)\ m$
- $(x_{w5},y_{w5})=(4632,2470)\ m$
- $K=100\ m/day$
- $N=30\ mm/year$
- $q=1.23\ m^2/day$
- $d=25\ m$



- $Q_{total}=20000\ m^3/day$
- $Q_{local\_min}(i)=200\ m^3/day$
- $Q_{local\_max}(i)=1500\ m^3/day$

Well's cone of depression:

- *radius of influence=300 m*
- *safety distance=100 m.*

Case 2

ALOPEX II algorithm:

$$Q(i)^{(k)}=Q(i)^{(k-1)}+c*[Q(i)^{(k-1)}-Q(i)^{(k-2)}]*[Profit^{(k-1)}-Profit^{(k-2)}]+noise^{(k)}$$

and Profit function:

$$Profit^{(k)}=\frac{\sum_{i=1}^{n} Q(i)^{(k)}}{\sum_{i=1}^{n} Q_{local\_max}(i)}$$ at k-th iteration.

Algorithm parameters:

- $c=0.6$: acceleration factor
- $noise(i)^{(k)}=0.05*Q(i)^{(k-1)}*(-0.5+1.5*rand)$.

Penalties management:

- Q_{local_min} penalty$=1.20$
- Q_{local_max} penalty$=0.95$
- *x-movement* penalty$=0.95$
- Q_{total} penalty$=1.20$
- ***interaction* penalty**

Case 2: Penalties definitions

- $\boldsymbol{Q_{local\_min}}$ ***penalty*** $\in[1,2]$

for i=1:n

  if $Q(i)^{(k)} \leq Q_{local\_min}(i)$

    $Q(i)^{(k)}=Q_{local\_min}\ penalty * Q(i)^{(k)}$

  end

end, at k-th iteration.

- $\boldsymbol{Q_{local\_max}}$ ***penalty*** $\in[0,1]$

for i=1:n

  if $Q(i)^{(k)} \geq Q_{local\_max}(i)$

    $Q(i)^{(k)}=Q_{local\_max}\ penalty * Q(i)^{(k)}$

  end

end, at k-th iteration.

- ***x-movement penalty*** $\in[0,1]$

for i=1:n

  if $x_T(i)^{(k)} \geq$ *safety-point(i)*

    $Q(i)^{(k)}=$ *x-movement penalty* $* Q(i)^{(k)}$

  end

end, at k-th iteration.

- $\boldsymbol{Q_{total}}$ ***penalty*** $\in[1,2]$

$S=\Sigma Q(i)^{(k)}$

for *i=1:n*

  $S1=S- Q(i)^{(k-1)}+ Q(i)^{(k)}$

  if $S1>Q_{total}$

    $Q(i)^{(k)} = Q(i)^{(k)} - Q_{total\_penalty} * (S1-Q_{total})$

  end

  S=S1;

end, at k-th iteration.

- ***interaction penalty***

Case 2

- ***interaction penalty*** P∈[0,1]

if i≠j and $x_{well}(i) >= x_{well}(j)$

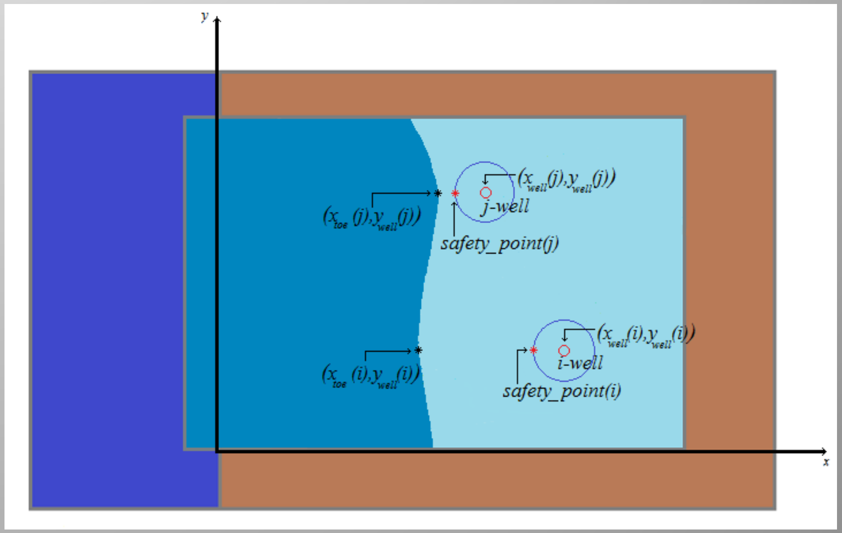
$$R(i,j)^{(k)} = \frac{Q_{local\_max}(i) - Q^{(k)}(i)}{\max\{Q_{local\_max}\}} * \frac{safety\_point(i) - x_{toe}^{(k)}(i)}{L} * \frac{\sqrt{[x_{well}(i) - x_{well}(j)]^2 + [y_{well}(i) - y_{well}(j)]^2}}{\sqrt{L^2 + W^2}}$$

else

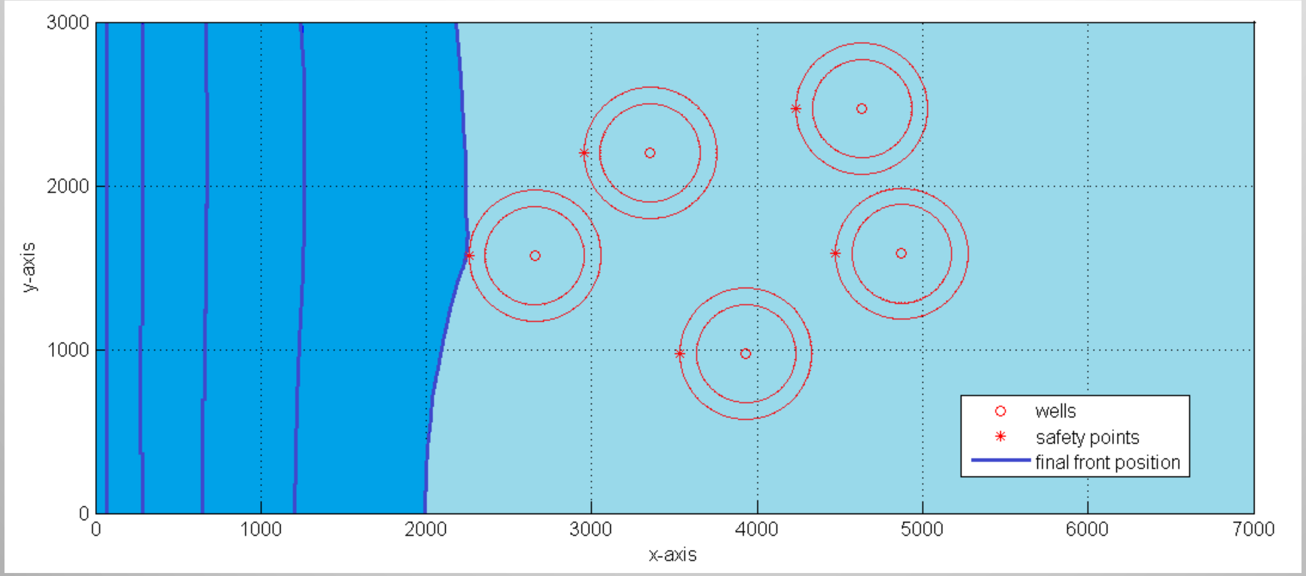
$$R(i,j)^{(k)} = 0.$$

So, the interaction penalty P that considers the coefficients $R(i,j)^{(k)}$ is activated using the following scheme:

if $R(i,j)^{(k)} = 0$: P = 1,
if $R(i,j)^{(k)} \in (0, 0.020)$: P = 0.98,
if $R(i,j)^{(k)} \in [0.020, 0.045)$: P = 0.96,
if $R(i,j)^{(k)} \in [0.045, 0.070)$: P = 0.92,
if $R(i,j)^{(k)} \in [0.070, 1]$: P = 0.88 .



Case 2: Numerical results using the MATLAB environment



Optimal values for pumping rates:

- $Q^{opt}(1)=768.90\ m^3/day$
- $Q^{opt}(2)=346.21\ m^3/day$
- $Q^{opt}(3)=1240.82\ m^3/day$
- $Q^{opt}(4)=1378.89\ m^3/day$
- $Q^{opt}(5)=206.12\ m^3/day$

$\Sigma Q^{opt}(i)=3934.95\ m^3/day.$

Penalties activation:

- Q_{local_min} penalty: 85 times
- Q_{local_max} penalty: 79 times
- *x-movement* penalty: 230 times
- *interaction* penalty: 174 times
- Q_{total} penalty: 0 times.

Discussion and conclusions

- The present work implements the ALOPEX optimization method to the problem of ***prevention of salination*** in freshwater aquifers.

- The study is based on a ***well known model*** of freshwater aquifers and its ***analytical solution*** for the ***water flow potential***. The ALOPEX method is chosen to calculate the optimal pumping rates of the aquifer wells, due to the ***advantages of the method*** when compared to other optimization tools.

- Simulations are presented for i) ***a hypothetical case*** of an aquifer with two wells and ii) ***the real aquifer case of Vathi*** (on the island of Kalymnos, Greece). A study on the sensitivity of the optimization process on the case of the aquifer of Vathi has also been performed, confirming the efficiency and applicability of the optimization method, as well as the need of the penalties imposed.

A few words about the ***advantages of ALOPEX optimization method***.

- by incorporating a ***stochastic element***, the method effectively finds the *global maxima* or *minima* without local convergence yet in a manner that does not require inefficient scanning for the solution.
- the ***profit function of the method is a scalar*** that measures global performance and can thus contain a large number of variables (related to the pumping rates of the aquifer wells), which may be simultaneously adjusted.
- the same optimization process can be applied in ***real time*** and is thus able to control the volume of pumping water in a ***real aquifer environment.***

- The method can be applied together with ***different pumping policies*** for the aquifer areas, giving us full control of the pumping management.

  For example, minimum and maximum pumping rates can differ for every well in controlling the volume of water distributed over the area. In this way, areas with different water needs (cities or agricultural areas) can be provided with no-less than the volume of water they actually need.

- No knowledge of the dynamics of the system or of the functional dependence of the cost function on the control variables, is required, making this way the method ***applicable*** to further, more ***realistic simulations where no analytical solutions are provided***.

Thank you for your attention,

Paris Stratis.