ALOPEX stochastic optimization for pumping management in fresh water coastal aquifers

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Εργαστήριο Εφαρμοσμένων Μαθηματικών
και Ηλεκτρονικών Υπολογιστών

ΘΑΛΗΣ

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ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΘΡΗΣΚΕΥΜΑΤΩΝ
ΕΙΔΙΚΗ ΥΠΗΡΕΣΙΑ ΔΙΑΧΕΙΡΙΣΗΣ
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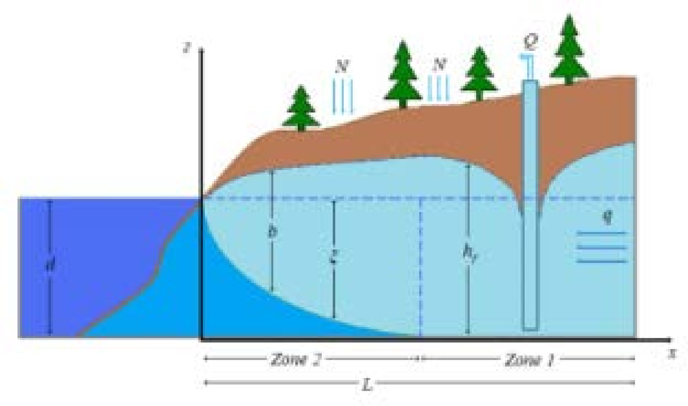
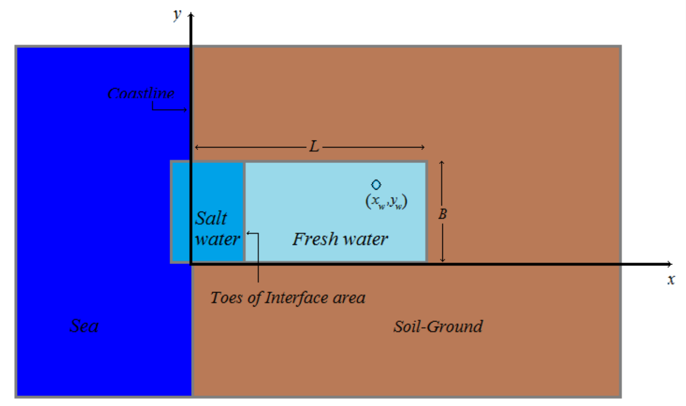
πρόγραμμα για την ανάπτυξη
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

Model equations

The model equations we use are based on the ***sharp interface approximation*** and the ***Ghyben-Herzberg relation***:

$$h_f - d = \delta\xi$$





The ***flow potential φ=φ(x,y)*** is a continuous and smooth function across the boundary between zones 1 and 2, satisfying the differential equation:

$$\frac{\partial}{\partial x}(K\frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y}(K\frac{\partial \phi}{\partial y}) + N - Q = 0$$

with *φ*=0 at the coastline (Dirichlet condition), $\frac{\partial \phi}{\partial n} = 0$ at the no-flow up and down impervious boundaries and fixed flow at the right side of the aquifer (Neumann conditions).

Q: pumping rates, *N*: rain factor and *K*: hydraulic conductivity.

Analytical solution

An analytical solution of this problem is the following:

$$\phi(x,y)=\frac{q}{K}x+\frac{N}{K}x(L-\frac{x}{2})+\sum_{k=1}^{2}\sum_{i=1}^{5}\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\ln(\frac{a_{i,j}(x)+b_{k,j}(y)}{a_{i+1,j}(x)+b_{k,j}(y)})+$$

$$+\sum_{n=1}^{2}\sum_{k=3}^{6}\sum_{i=1}^{5}\sum_{j=1}^{M}\frac{Q_j}{4\pi K}\ln(\frac{a_{i,j}(x)+b_{k,n,j}(y)}{a_{i+1,j}(x)+b_{k,n,j}(y)})$$

where:

$$a_{1,j}(x):=(x-x_j)^2 \qquad b_{1,j}(y):=(y-y_j)^2$$

$$a_{2,j}(x):=(x+x_j)^2 \qquad b_{2,j}(y):=(y+y_j)^2$$

$$a_{3,j}(x):=(x-(2L-x_j))^2 \qquad b_{3,n,j}(y):=(y-(2nB-y_j))^2$$

$$a_{4,j}(x):=(x-(2L+x_j))^2 \qquad b_{4,n,j}(y):=(y-(2nB+y_j))^2$$

$$a_{5,j}(x):=(x+(2L+x_j))^2 \qquad b_{5,n,j}(y):=(y+(2nB-y_j))^2$$

$$a_{6,j}(x):=(x+(2L-x_j))^2 \qquad b_{6,n,j}(y):=(y+(2nB+y_j))^2$$

(x_j,y_j): the coordinates of the j^{th} aquifer well, $j\in\{1,2,\ldots,M\}$, L: length and B: width of the aquifer.

Optimization procedure

maximize: $$P = P(Q_1, Q_2, \ldots, Q_M) = e^{-\left(1 - \frac{\sum_{j=1}^{M} Q_j}{\sum_{j=1}^{M} \overline{Q_j}}\right)^2}$$

s.t.: $$\underline{Q_j} \le Q_j \le \overline{Q_j}$$

$$\sum_{j=1}^{M} Q_j \le Q_A$$

$$x_{\tau,j} \le x_j - d_s, \quad j \in \{1, 2, \ldots, M\}$$

where:

$\overline{Q_j}, \underline{Q_j}$: maximum and minimum pumping capabilities of the *j*th aquifer well per day

Q_A: total pumping capabilities of the aquifer/day

d_s: radius of the safety distance around every well

$x_{\tau,j}$: x-coordinate of the ***interface toes area*** opposite the *j*th aquifer well.

ALOPEX

ALgorithm **O**f **P**attern **EX**traction is a stochastic iterative procedure, used to produce the new pumping rates in every iteration step.

$$Q_j^{(k)} = Q_j^{(k-1)} + c\left(Q_j^{(k-1)} - Q_j^{(k-2)}\right)\frac{\left(P^{(k-1)} - P^{(k-2)}\right)}{\left|P^{(k-1)} - P^{(k-2)}\right|} + g_j^{(k)}, \quad j \in \{1, \ldots, M\}$$

where:

c: acceleration factor and $g_j^{(k)}$: white noise for the *j*th well at *k*th iteration.

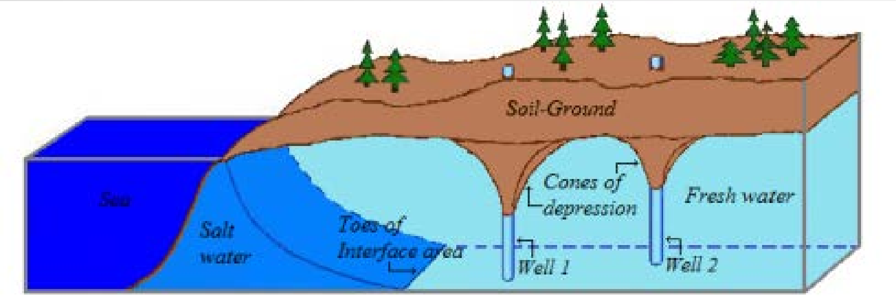
Numerical simulations (part 1)

We examine the following examples of a test case aquifer, with geometry and specific properties that approximate the aquifer at Vathi area at the Greek island of Kalymnos.

Rectangular aquifer:

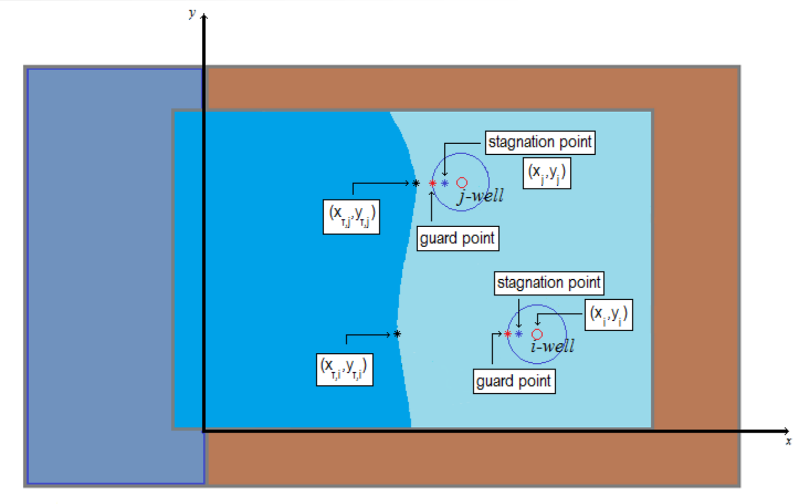
L=7000 m
W=3000 m
K=100 m/day
N_{upper}=150 mm/year
N_{lower}=30 mm/year

Safety strategy



The saltwater front is moving towards the ***cone of depression areas*** of the aquifer wells.

In order to preserve the ***stagnation points*** of the pumping procedure, we choose a number of guard points in front of every aquifer well. In this way, a safety zone (with a total radius equal to d_s) of no saltwater intrusion is created around every pumping location, leading to stable pumping solutions.



ALOPEX pumping setup

ALOPEX V:

$$Q_j^{(k)} = Q_j^{(k-1)} + c\left(Q_j^{(k-1)} - Q_j^{(k-2)}\right)\frac{\left(P^{(k-1)} - P^{(k-2)}\right)}{\left|P^{(k-1)} - P^{(k-2)}\right|} + g_j^{(k)}, \qquad j \in \{1, \ldots, M\}$$

where:

acceleration factor *c=1* and white noise $g_j^{(k)} = 0.015\, Q_j^{(k-1)} (0.5+1.5*rand)$ for the j^{th} well at k^{th} iteration.

Specific aquifer properties:

$$\overline{Q}_j = 2500.00 \quad (\mathrm{m}^3/\mathrm{day})$$

$$\underline{Q}_j = 200.00 \quad (\mathrm{m}^3/\mathrm{day})$$

$$Q_A = 20000 \quad (\mathrm{m}^3/\mathrm{day})$$

Penalties management:

Qlocal_max penalty = 0.95

Qlocal_min penalty = 1.05

Qtotal penalty = 1.05

x_movement penalty = 0.95

critical_distance penalty = 0.95

Safety distance:

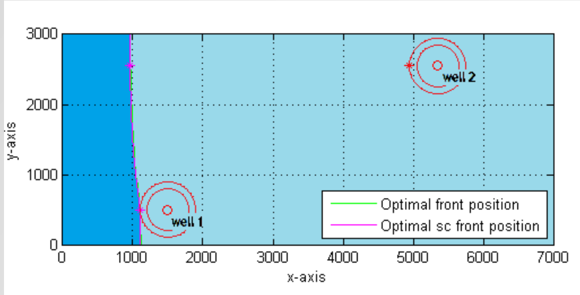
$$d_s = 400\ m$$

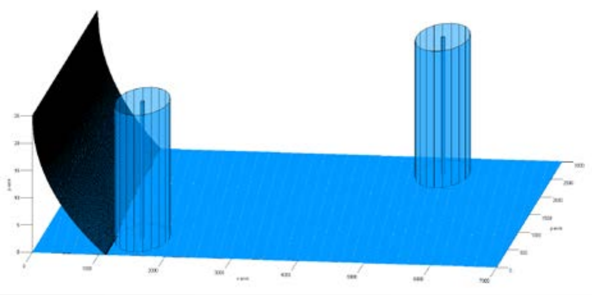
Stopping criterion:

$$mean\left|P^{(k-20:k)} - P^{(k-40:k-20)}\right| < 0.01$$

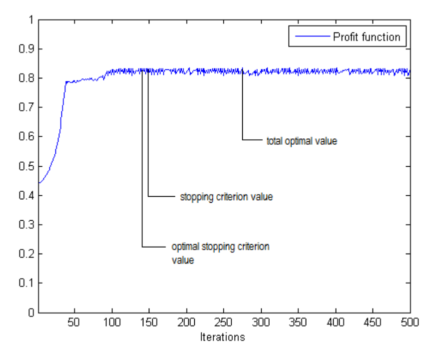
$$std\left|P^{(k-40:k)}\right| < 0.01$$

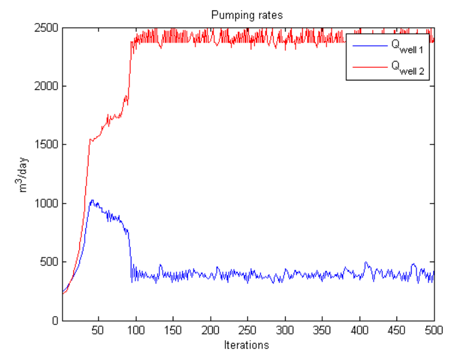
ALOPEX-Analytical solution case Ia, aquifer with 2 wells





- Iterations: 500
- Time: 1.394613 seconds





Number of activations of the:

- Qlocal_max penalty: 142
- Qlocal_min penalty: 0
- Qtotal penalty: 0
- x_movement penalty: 142
- critical_distance penalty: 34

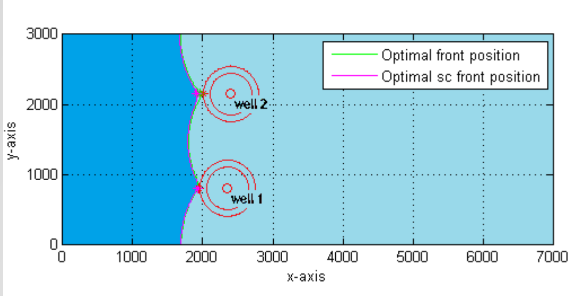
Optimal pumping rates iteration number : 275

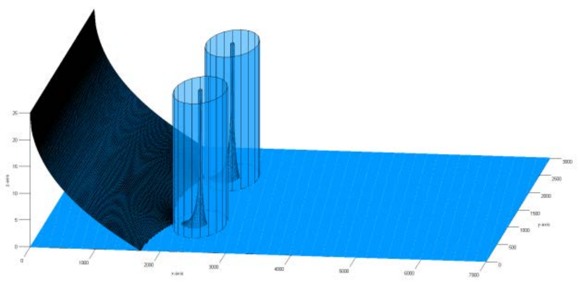
- Pumping rates (cubic meters/day): 371.39 2499.44
- Sum (cubic meters/day): 2870.83

Stopping criterion optimal pumping rates iteration number : 146

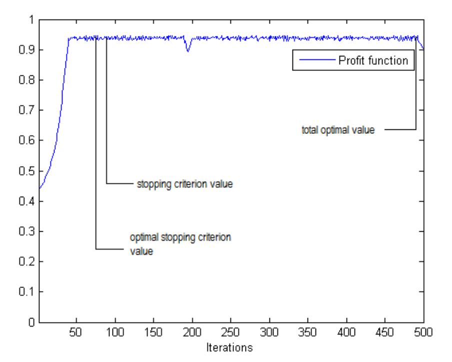
- Pumping rates (cubic meters/day): 389.53 2472.49
- Sum (cubic meters/day): 2862.02

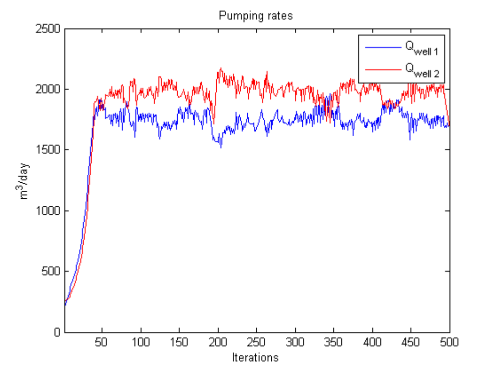
ALOPEX-Analytical solution case Ib, aquifer with 2 wells





- Iterations: 500
- Time: 1.969666 seconds





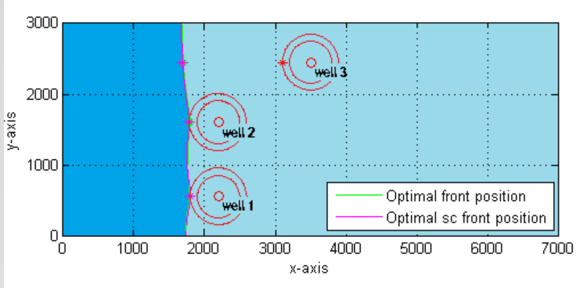
Number of activations of the:

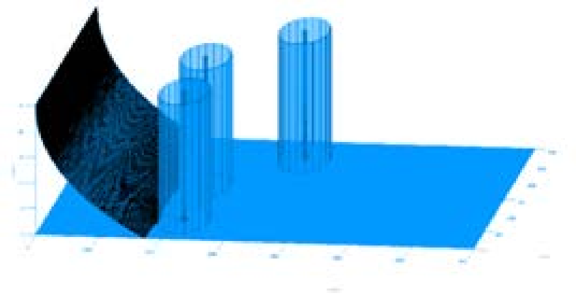
- Qlocal_max penalty: 0
- Qlocal_min penalty: 0
- Qtotal penalty: 0
- x_movement penalty: 351
- critical_distance penalty: 7

Optimal pumping rates iteration number : 492
- Pumping rates (cubic meters/day): 1783.52 2038.97
- Sum (cubic meters/day): 3822.49

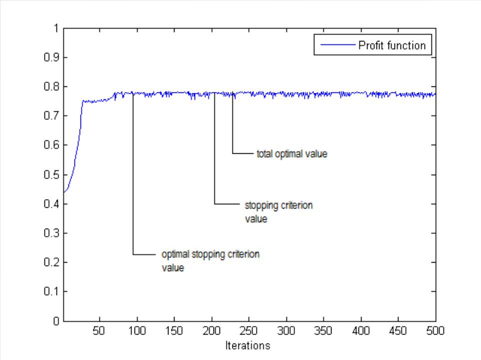
Stopping criterion optimal pumping rates iteration number : 75
- Pumping rates (cubic meters/day): 1793.04 2024.97
- Sum (cubic meters/day): 3818.01

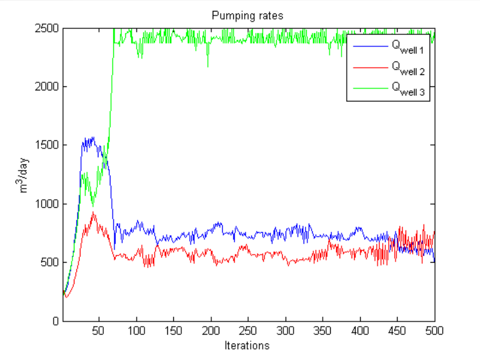
ALOPEX-Analytical solution case IIa, aquifer with 3 wells





- Iterations: 500
- Time: 2.279823 seconds





Number of activations of the:

- Qlocal_max penalty: 133
- Qlocal_min penalty: 1
- Qtotal penalty: 0
- x_movement penalty: 206
- critical_distance penalty: 13

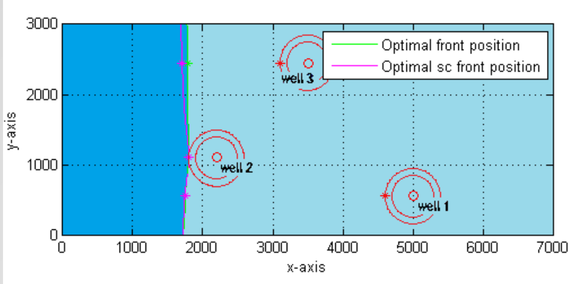
Optimal pumping rates iteration number : 228

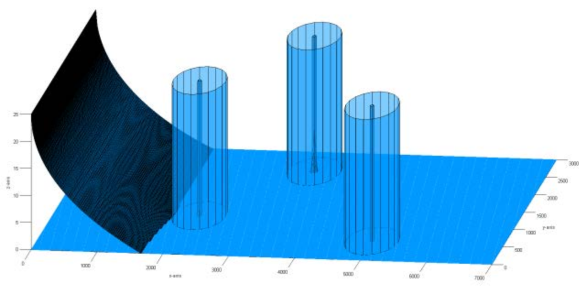
- Pumping rates (cubic meters/day): 752.67 548.25 2488.18
- Sum (cubic meters/day): 3789.11

Stopping criterion optimal pumping rates iteration number : 94

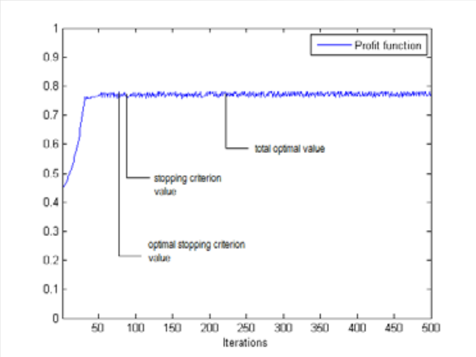
- Pumping rates (cubic meters/day): 769.21 530.57 2486.10
- Sum (cubic meters/day): 3785.88

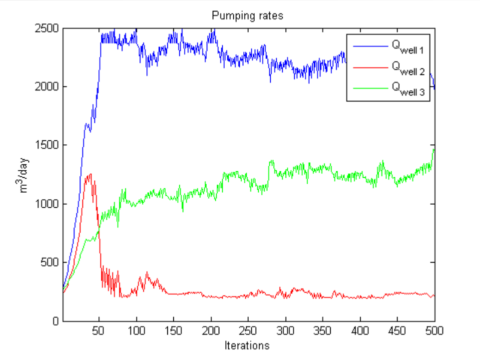
ALOPEX-Analytical solution case IIb, aquifer with 3 wells





- Iterations: 500
- Time: 3.481994 seconds





Number of activations of the:

- Qlocal_max penalty: 33
- Qlocal_min penalty: 24
- Qtotal penalty: 0
- x_movement penalty: 242
- critical_distance penalty: 372

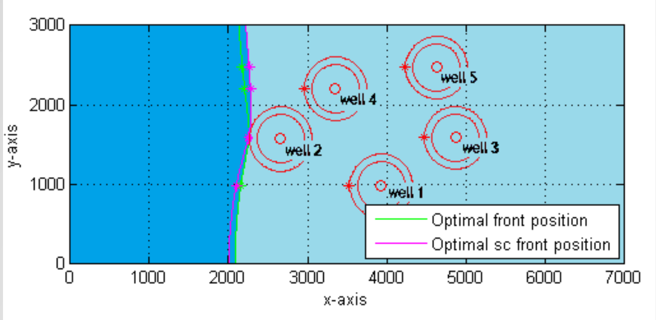
Optimal pumping rates iteration number : 221

- Pumping rates (cubic meters/day): 2306.97 206.63 1254.18
- Sum (cubic meters/day): 3767.78

Stopping criterion optimal pumping rates iteration number : 77

- Pumping rates (cubic meters/day): 2443.32 293.57 1004.77
- Sum (cubic meters/day): 3741.67

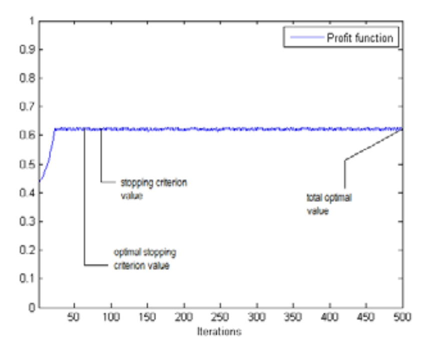
ALOPEX-Analytical solution case III, aquifer with 5 wells

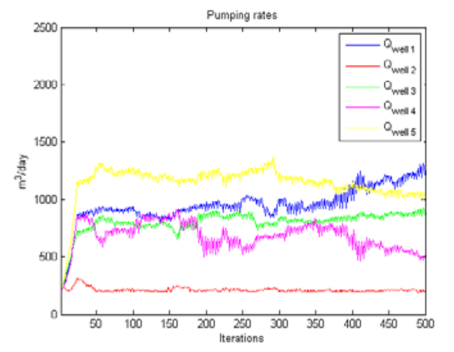


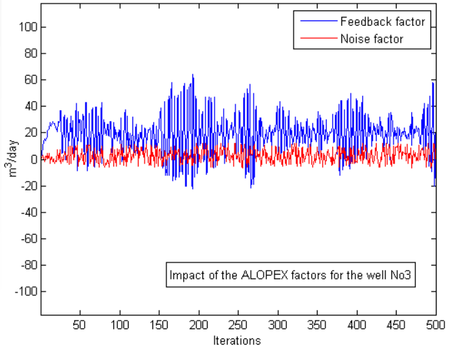
- Iterations: 500
- Time: 7.870688 seconds

Number of activations of the:

- Qlocal_max penalty: 0
- Qlocal_min penalty: 52
- Qtotal penalty: 0
- x_movement penalty: 324
- critical_distance penalty: 993







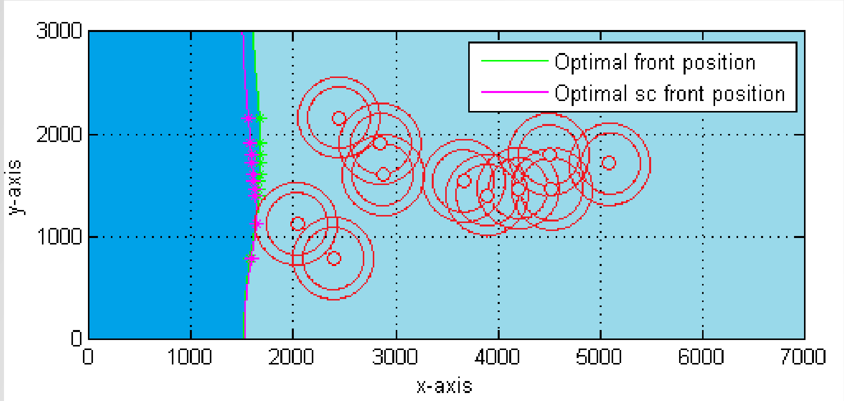
Optimal pumping rates iteration number : 500

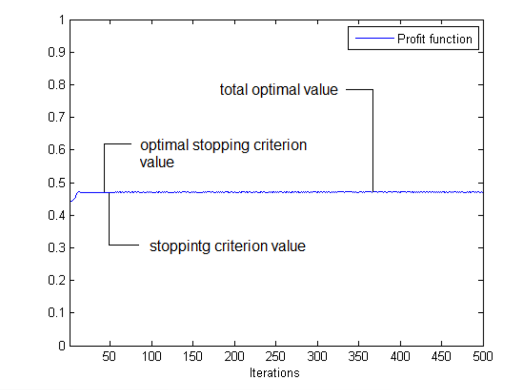
- Pumping rates (cubic meters/day): 1303.05 202.27 912.85 504.38 1047.07
- Sum (cubic meters/day): 3969.62

Stopping criterion optimal pumping rates iteration number : 63

- Pumping rates (cubic meters/day): 923.57 206.41 838.96 720.10 1278.27
- Sum (cubic meters/day): 3967.30

ALOPEX-Analytical solution case IV, aquifer with 11 wells





- Iterations: 500
- Time: 17.357844seconds

Number of activations of the:

- Qlocal_max penalty: 0
- Qlocal_min penalty: 157
- Qtotal penalty: 0
- x_movement penalty: 365
- critical_distance penalty: 2740

Optimal pumping rates iteration number : 364

- Pumping rates (cubic meters/day): 207.41 200.11 205.80 623.30 413.77 286.40 345.26 223.07 207.62 365.96 597.52
- Sum (cubic meters/day): 3676.22

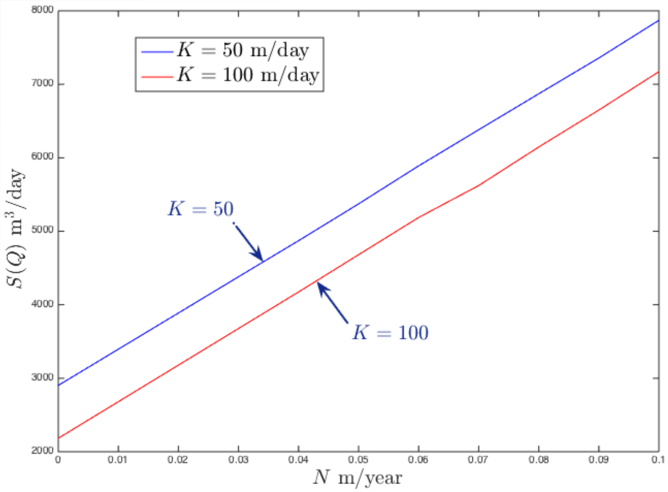
Stopping criterion optimal pumping rates iteration number : 47

- Pumping rates (cubic meters/day): 923.57 206.41 838.96 302.80 314.07 461.57 278.70 315.99 224.3168 283.75 284.63 330.94 439.21 400.27
- Sum (cubic meters/day): 3636.24

Some remarks (Kalymnos aquifer)

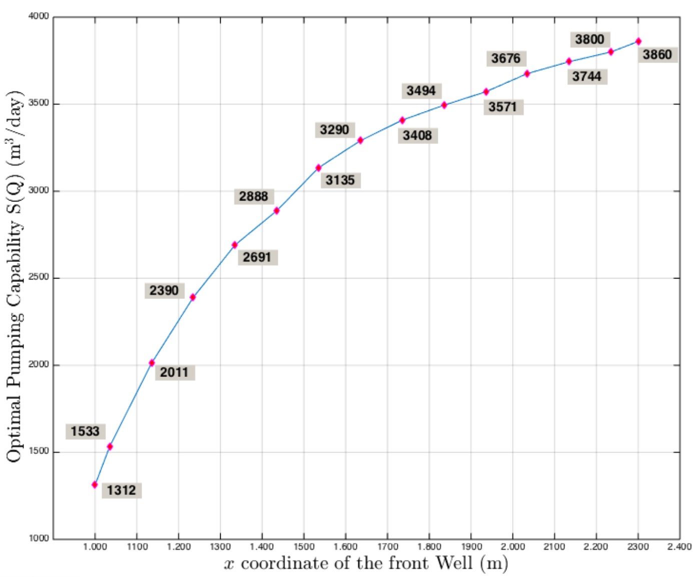
- ALOPEX optimization procedure can provide us with an effective and quite fast pumping management setup of all the aquifer wells.
- ALOPEX algorithm needs only a few iterations in order to converge to an optimal solution, by using an appropriate stopping criterion.
- In order to further exploit the dependence of the reported results on the values of the hydraulic conductivity *K* and the recharge rate *N*, we present the next Figure.

  It becomes apparent the linear dependence of the pumping rate *S(**Q**)* on the aquifer's recharge rate *N*, for fixed values of the hydraulic conductivity *K*.



Some remarks (Kalymnos aquifer)

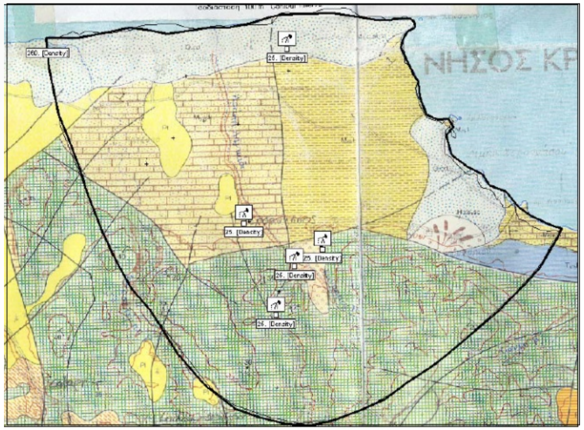
- We also present the dependence of the total optimal pumping rate on the distance of the front well from the coastline, for fixed values of the hydraulic conductivity *K* and the recharge rate *N*.



Numerical simulations (part 2)

We also examine the case of a freshwater aquifer located at the Municipality of Hersonissos, 26km east of Heraklion, at the Greek island of Crete.

The aquifer covers an area of almost 16km^2 .



Princeton Transport Code

PTC is a hybrid 3D model using the Finite Elements method in order to represent the **groundwater flow** and the **contaminant transport** in a freshwater coastal aquifer.

In the present work we evaluate **only the groundwater flow** inside the aquifer, in respect to the hydraulic head h:

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)-S\frac{\partial h}{\partial t}+\sum_{i=1}^{n}Q_i\delta(x-x_i)\delta(y-y_i)\delta(z-z_i)=0$$

with groundwater velocity components:

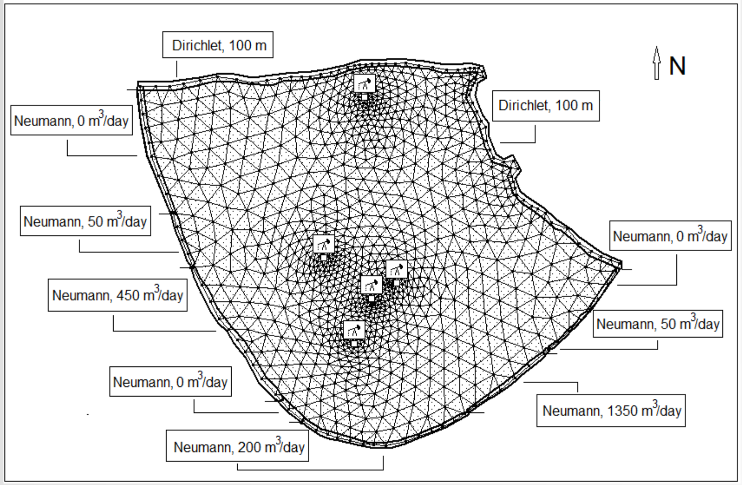
$$V_x = \text{-K}_{xx}\frac{\partial h}{\partial x},\ V_y = \text{-K}_{yy}\frac{\partial h}{\partial y},\ V_z = \text{-K}_{zz}\frac{\partial h}{\partial z}$$

where:

- h is the hydraulic head (m)
- K_{xx} is the hydraulic conductivity in the x (horizontal) direction (m/day),
- K_{yy} is the hydraulic conductivity in the y (horizontal) direction (m/day),
- K_{zz} is the hydraulic conductivity in the z (vertical) direction (m/day),
- S is the specific storage coefficient (day^{-1}),
- Q_i is the pumping rate of the i^{th} well of the aquifer (m^3/day),
- δ is the Dirac delta function,
- n is the number of aquifer wells.

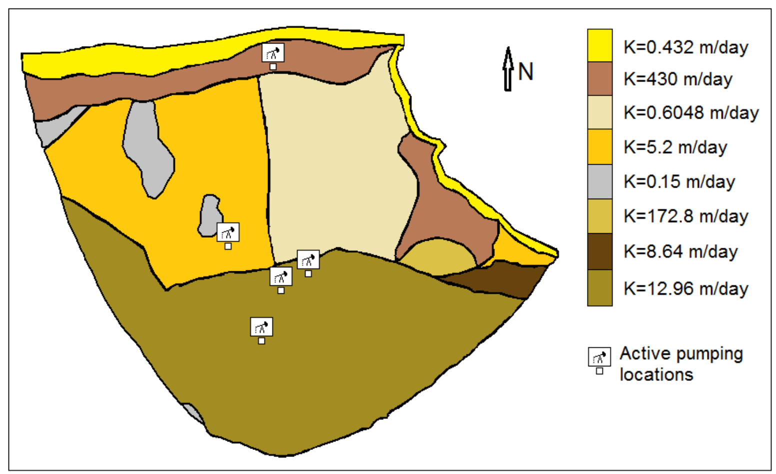
The above equations are derived from conservation of mass principles and Darcy's Law.

Hersonissos aquifer
Boundary conditions and Hydraulic conductivities

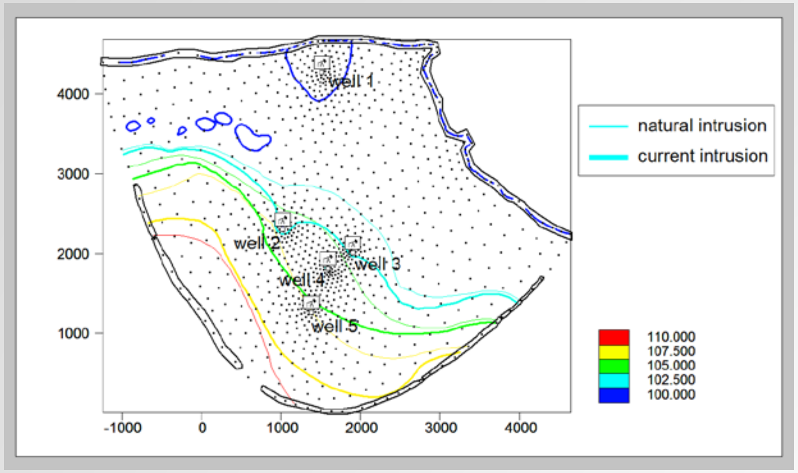


PTC creates a mesh of 1050 nodes and 1984 triangular elements with Dirichlet conditions at the sea area and Neumann conditions at the inland of the aquifer.

Inside the aquifer there exist areas of different hydraulic conductivities K, as represented in the next figure.



Natural intrusion and previous work

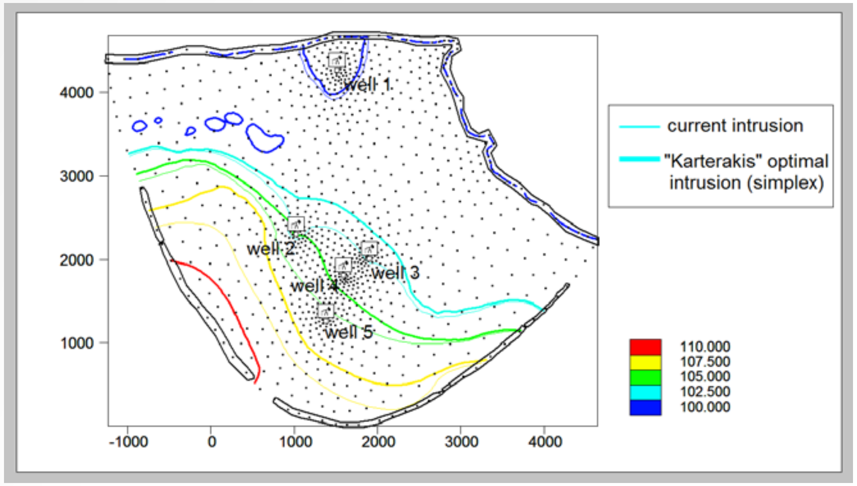


Natural intrusion when all aquifer wells are inactive and ***current intrusion***, while the municipality of Hersonissos is using the following summer pumping setup:

- Pumping rates (cubic meters/day): 1800.00 2520.00 576.00 2520.00 146.00
- Sum (cubic meters/day): 7562.00

Current intrusion with the pumping setup presented above vs the optimal pumping setup presented by *Karterakis et al (2005)*, using the ***simplex method***:

- Pumping rates (cubic meters/day): 1800.00 631.50 576.00 1935.40 0.00
- Sum (cubic meters/day): 4942.90



ALOPEX pumping setup

ALOPEX V:

$$Q_j^{(k)} = Q_j^{(k-1)} + c\left(Q_j^{(k-1)} - Q_j^{(k-2)}\right)\frac{\left(P^{(k-1)} - P^{(k-2)}\right)}{\left|P^{(k-1)} - P^{(k-2)}\right|} + g_j^{(k)}, \qquad j \in \{1, \dots, M\}$$

where:

acceleration factor *c=1* and white noise $g_j^{(k)} = 0.015\, Q_j^{(k-1)}\, (0.5+1.5*rand)$ for the *j*th well at *k*th iteration.

Specific aquifer properties:

$$\overline{Q} = 1800.00 \quad 2520.00 \quad 576.00 \quad 2520.00 \quad 146.00 \qquad (\mathrm{m}^3/\mathrm{day})$$

$$\underline{Q} = 0.30\overline{Q} = 540.00 \quad 756.00 \quad 178.80 \quad 756.00 \quad 43.80 \qquad (\mathrm{m}^3/\mathrm{day})$$

$$Q_A = 10000 \qquad (\mathrm{m}^3/\mathrm{day})$$

Penalties management:

Qlocal_max penalty = 0.95

Qlocal_min penalty = 1.05

Qtotal penalty = 1.05

x_movement penalty = 0.95

critical_distance penalty = 0.95

Safety distance:

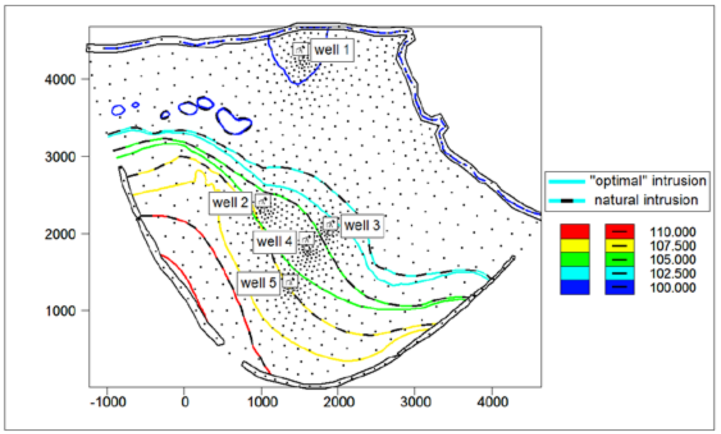
$$d_s = 200\ m$$

Stopping criterion:

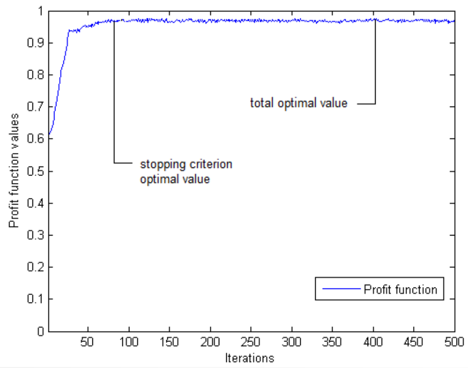
$$mean\left|P^{(k-20:k)} - P^{(k-40:k-20)}\right| < 0.01$$

$$std\left|P^{(k-40:k)}\right| < 0.01$$

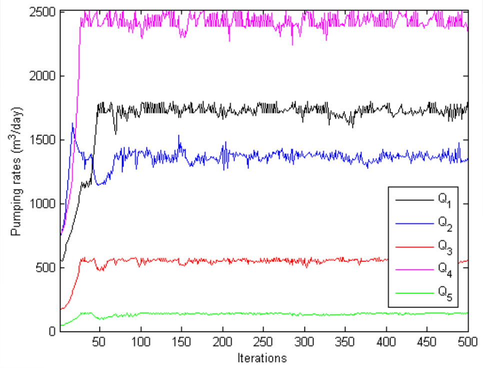
ALOPEX-PTC case I
summer conditions, aquifer with 5 wells



Saltwater front inside the aquifer where stands : $h = 102.5m$



- Iterations: 500
- Time: 485.13 seconds



Number of activations of the:

- Qlocal_max penalty: 373
- Qlocal_min penalty: 2
- Qtotal penalty: 0
- x_movement penalty: 81
- critical_distance penalty: 69

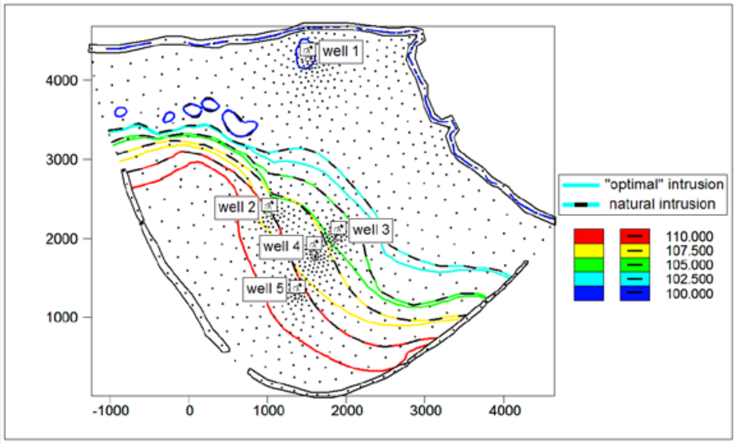
Optimal pumping rates iteration number: 440

- Pumping rates (cubic meters/day): 1710.00 1652.19 557.41 2342.64 136.60
- Sum (cubic meters/day): 6398.84

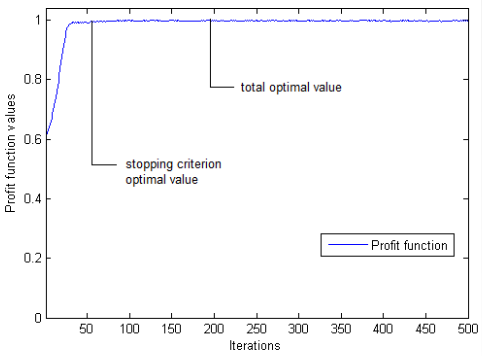
Stopping criterion pumping rates iteration number: 60

- Pumping rates (cubic meters/day): 1622.01 1339.13 551.69 2503.90 120.87
- Sum (cubic meters/day): 6137.61

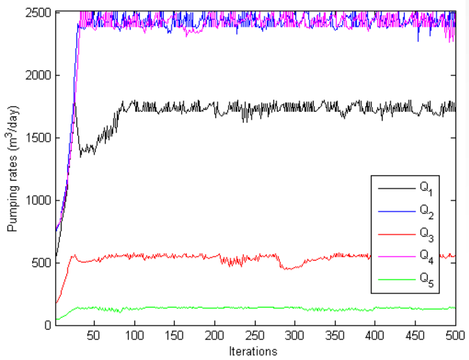
ALOPEX-PTC case II, winter conditions, aquifer with 5 wells



Saltwater front inside the aquifer where stands : $h = 102.5m$



- Iterations: 500
- Time: 580.55 seconds



Number of activations of the:

- Qlocal_max penalty: 240
- Qlocal_min penalty: 2
- Qtotal penalty: 0
- x_movement penalty: 109
- critical_distance penalty: 9

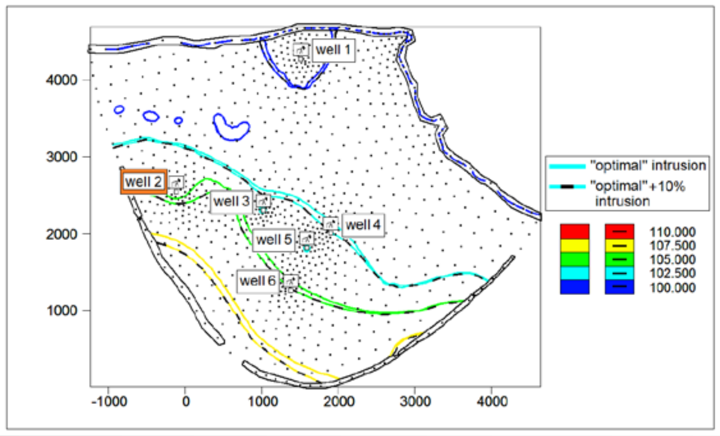
Iteration number of the optimal pumping rates: 198

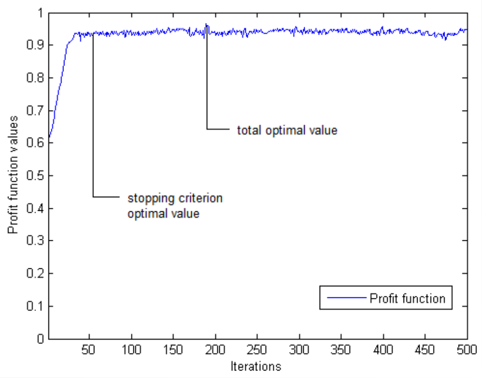
- Optimal pumping rates (cubic meters/day): 1777.16 2514.42 545.27 2511.35 136.80
- Sum of the optimal pumping rates (cubic meters/day): 7484.99

Iteration number of the stopping criterion pumping rates: 64

- Stopping criterion pumping rates (cubic meters/day): 1457.85 2434.25 554.24 2394.00 134.72
- Sum of the stopping criterion pumping rates (cubic meters/day): 6975.06

ALOPEX-PTC case IIIa, summer conditions, aquifer with 6 wells





- Iterations: 500
- Time: 893.05 seconds

Number of activations of the:

- Qlocal_max penalty: 147
- Qlocal_min penalty: 32
- Qtotal penalty: 0
- x_movement penalty: 255
- critical_distance penalty: 688

Saltwater front inside the aquifer where stands : $h = 102.5m$

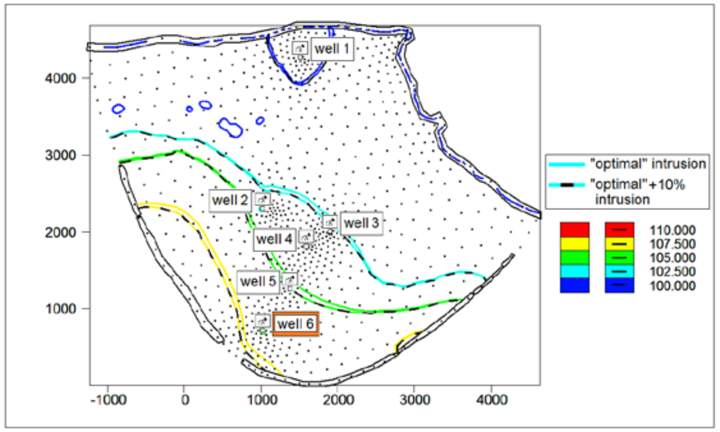
Optimal pumping rates iteration number: 189

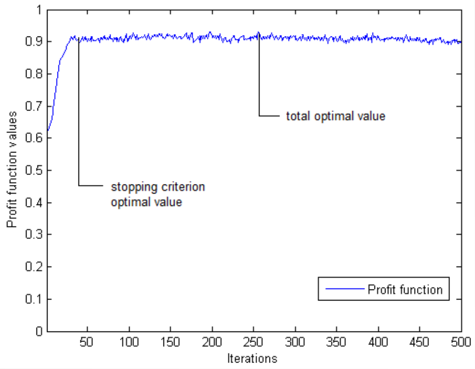
- Pumping rates (cubic meters/day): 1710.00 2499.29 1099.91 227.50 2504.27 114.33
- Sum (cubic meters/day): 8155.31

Stopping criterion pumping rates iteration number: 69

- Pumping rates (cubic meters/day): 1774.93 2070.60 1021.42 239.95 2265.70 135.09
- Sum (cubic meters/day): 7507.69

ALOPEX-PTC case IIIb,
summer conditions, aquifer with 6 wells





- Iterations: 500
- Time: 978.42 seconds

Number of activations of the:

- Qlocal_max penalty: 86
- Qlocal_min penalty: 62
- Qtotal penalty: 0
- x_movement penalty: 258
- critical_distance penalty: 788

Saltwater front inside the aquifer where stands : $h = 102.5m$

Optimal pumping rates iteration number: 256

- Pumping rates (cubic meters/day): 1710.00 1263.83 181.44 1728.71 87.26 2393.37
- Sum (cubic meters/day): 7364.61

Stopping criterion pumping rates iteration number: 63

- Pumping rates (cubic meters/day): 1710.00 1243.65 181.44 1711.83 83.23 2054.26
- Sum (cubic meters/day): 6984.41

Some remarks (Hersonissos aquifer)

- ALOPEX optimization procedure can provide us with an effective and quite fast pumping management setup of all the aquifer wells.

- ALOPEX algorithm needs only a few iterations in order to converge to an optimal solution, by using an appropriate stopping criterion.

- Well No 1 is constructed in a wrong place, very close to the sea interface. It is saltwater flooded even when all the aquifer wells are inactive. So, it should not be included in the pumping management procedure and be left inactive.

- The safety distance around every aquifer well is set to be equal to 200m. This fact creates a safety margin of almost 10% to increase the optimal pumping water volume, without risking the saltwater integrity of all the aquifer wells.

- The freshwater basin of the Hersonissos aquifer is able to provide a larger amount of water the local community, by increasing the number of the active aquifer wells. We presented examples with only one extra well (see cases with 6 wells), where the total amount of pumping water was significantly increased.

Publications

- P. Stratis, Y. Saridakis, E. Papadopoulou and M. Zakynthinaki, *ALOPEX Stochastic Optimization for Pumping Management in Freshwater Coastal Aquifers,* Journal of Physics: Conference Series, 490, 012112, 2014.

- P. Stratis, Z. Dokou, G. Karatzas, E. Papadopoulou and Y. Saridakis, *Stochastic Optimization and Numerical Simulation for Pumping Management of the Hersonissos Freshwater Coastal Aquifer in Crete,* Procs. of INASE/CSCCWHH 2015, Recent Advances in Environmental and Earth Sciences and Economics, 329-334, Zakynthos, 2015.

- P. Stratis, Z. Dokou, G. Karatzas, E. Papadopoulou and Y. Saridakis, *PTC Simulations, Stochastic Optimization and Safety Strategies for Freshwater Pumping Management: Case Study of the Hersonissos Coastal Aquifer in Crete,* September 2015,
  (Applied Water Sciences submitted).

- P. Stratis, G. Karatzas, E. Papadopoulou, M. Zakynthinaki and Y. Saridakis, *Stochastic Optimization for an Analytical Model of Saltwater Intrusion in Coastal Aquifers,* 2015
  (PLOSone submitted).

- I. Athanasakis, Z. Dokou, E. Mathioudakis, P. Stratis and N. Vilanakis, *Combining Stochastic Optimization and Numerical Methods-Software for the Pumping Management of Coastal Aquifers: Case Study of a Rectangular Homogeneous Aquifer,* Conference in Mathematical Methods and Computational Techniques in Science and Engineering, November 28-30, Bratislava, Slovakia, 2015.

Bibliography

[1] R. Ababou and A. Al-Bitar, Salt water intrusion with heterogeneity and uncertainty: mathematical modeling and analyses, Developments Water Sci., 55:1559-1571, 2004.

[2] M. Aivalioti and G. Karatzas, Modeling the flow and leachate transport in the vadose and saturated Zone - A field application, Env Model Assess., 11(1):81-87, 2006.

[3] D. Babu, G. Pinder, A. Niemi, D. Ahlfeld and S. Stothoff, Chemical transport by three-dimensional groundwater flows, Princeton University, 84-WR-3, USA, 1997.

[4] Z. Dokou and G. Karatzas, Saltwater intrusion estimation in a karstified coastal system using density-dependent modeling and comparison with the sharp-interface approach, Hydrol. Sci., J 57(5):985-999, 2012.

[5] Z. Dokou and G. Pinder, Extension and field application of an integrated DNAPL source identification algorithm that utilizes stochastic modeling and a Kalman filter, J. Hydrol., 398(3-4):277-291, 2011.

[6] V. Guvanase, S. Wade and M. Barcelo, Simulation of regional ground water flow and salt water intrusion in Hernando County, Florida, Ground Water, 38(5):772-783, 2000.

[7] C. W. Fetter, Applied Hydrogeology, Merrill Publishing Company, 1988.

[8] E. Harth and E. Tzanakou, Alopex: A stochastic method for determining visual receptive fields, Vision Research, 14, pp.1475, B1482, 1974.

[9] G. Karatzas and Z. Dokou, Managing the saltwater intrusion phenomenon in the coastal aquifer of Malia, Crete using multi-objective optimization, Hydrogeology, 2015, accepted.

[10] S. Karterakis, G. Karatzas, I. Nikolos and M. Papadopoulou Application of linear programming and differential evolutionary optimization methodologies for the solution of coastal subsurface water management problems subject to environmental criteria, J. Hydrol., 342(3-4):270-282, 2007.

[11] M. Koukadaki, G. Karatzas, M. Papadopoulou and A. Vafidis, Identification of the saline zone in a coastal aquifer using electrical tomography data and simulation, Water Resour. anag., 21(11):1881-1898, 2007.

Bibliography

[12] A. Mantoglou, Pumping management of coastal aquifers using analytical models of saltwater intrusion, Water Resources Research, ISSN 0043-397, 39(12), 2003.

[13] A. Mantoglou and M. Papantoniou, Optimal Design Of Pumping Networks In Coastal Aquifers Using Sharp Interface Models, J. Hydrol., 361:52-63, 2008.

[14] A. Mantoglou, M. Papantoniou and P. Giannoulopoulos, Management of coastal aquifers based on nonlinear optimization and evolutionary algorithms, J. Hydrol., 297(1-4):209-228, 2004.

[15] M. Papadopoulou, E. Varouchakis and G. Karatzas, Simulation of complex aquifer behavior using numerical and geostatistical methodologies, Desalination, 237:42-53, 2009.

[16] M. Papadopoulou, E. Varouchakis and G. Karatzas, Terrain Discontinuity Effects in the Regional Flow of a Complex Karstified Aquifer, Environ. Model Assess, 15(5):319-328, 2010.

[17] O.D.L. Strack, Groundwater Mechanics, Prentice Hall, 1989.

[18] P. Stratis, G. Karatzas, E. Papadopoulou, M. Zakynthinaki and Y. Saridakis, Stochastic optimization for an analytical model of saltwater intrusion in coastal aquifers, 2015, (PLOSone submitted).

[19] T. Reilly and A. Goodman, Quantitative-Analysis of Saltwater Fresh-Water Relationships in Groundwater Systems – a Historical-Perspective, J. Hydrol., 80(1-2):125-160, 1985.

[20] K. Voudouris, D. Mandilaras and A. Antonakos, Methods to define the areal distribution of the salt intrusion: Examples from South Greece, 18 SWIM. Cartagena, Spain. (Ed. Aragus, Custod Io and Manzano), 2004.

[21] M. Zakynthinaki and Y. Saridakis, Stochastic optimization for a tip-tilt adaptive correcting system, Comp. Phys. Commun., 150(3) 274, 2003.

Thank you for your attention,

Paris N. Stratis.