

ALOPEX stochastic optimization for pumping management in fresh water coastal aquifers

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Abstract. Saltwater intrusion in freshwater aquifers is a problem of increasing significance in areas nearby the coastline. Apart from natural disastrous phenomena, such as earthquakes or floods, intense pumping human activities over the aquifer areas may change the chemical composition of the freshwater aquifer. Working towards the direction of real time management of freshwater pumping from coastal aquifers, we have considered the deployment of the stochastic optimization *Algorithm of Pattern Extraction* (ALOPEX), coupled with several penalty strategies that produce convenient management policies. The present study, which further extends recently derived results, considers the analytical solution of a classical model for underground flow and the ALOPEX stochastic optimization technique to produce an efficient approach for pumping management over coastal aquifers. Numerical experimentation also includes a case study at Vathi area on the Greek island of Kalymnos, to compare with known results in the literature as well as to demonstrate different management strategies.

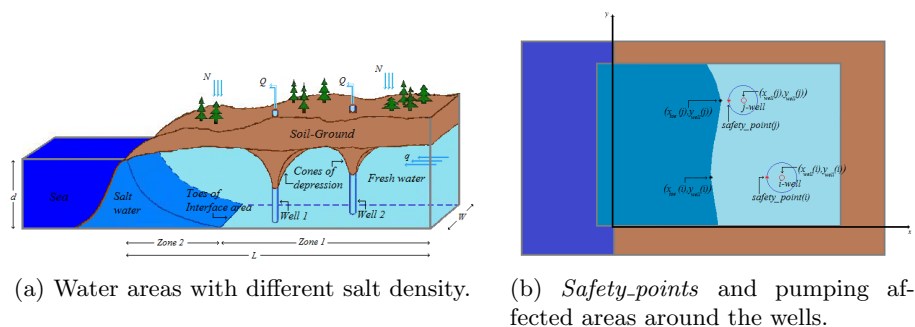


Figure 1: Coastal freshwater aquifer.

1. ALOPEX II optimization

In the present study a well-known existing mathematical model, presented in [2] and [3], is numerically simulated and the management of freshwater pumping is investigated and optimized by means of stochastic optimization. Extending our results, recently included in [4], a new, well established (e.g. [2]), profit function is used and, although it is a linear one, the convergence of the stochastic optimization ALOPEX II algorithm ([1], [5]) is achieved in a few iterations. Furthermore, the penalties management setup, proposed in [4], is being qualitatively improved to a management procedure that effectively controls well pumping rates taking into consideration wells' positions and their pumping capabilities.



1.1. Toe constraint formulation

The problem at hand is the determination of the optimal values for the pumping rates $Q(i)$, $i = 1, \dots, n$, with n being the number of wells, without contaminating with saltwater any one of the wells pumping fresh water over the aquifer. This problem is formulated as follows:

$$\begin{aligned} & \text{maximize Profit}(Q_1, \dots, Q_N) = \frac{\sum_{i=1}^n Q(i)}{\sum_{i=1}^n Q_{local}^{max}(i)} \leq 1 \\ & \text{under the constrains} \\ & x_{well}(i) - x_{toe}(i) \geq D, \quad 0 \leq Q_{local}^{min}(i) \leq Q(i) \leq Q_{local}^{max}(i), \quad \sum_{i=1}^n Q(i) \leq Q_{aquifer}^{total}, \end{aligned} \quad (1)$$

where $x_{toe}(i)$ denotes the x-coordinate of the saltwater interface opposite to the i -well and $D = x_{well}(i) - safety_point(i) > 0$ is a safety distance (see also figure 1b). The new values of pumping rates $Q(i)$ are determined at every ALOPEX II iteration step, using the following iteration scheme:

$$Q^{(k)}(i) = Q^{(k-1)}(i) + c[Q^{(k-1)}(i) - Q^{(k-2)}(i)][Profit^{(k-1)} - Profit^{(k-2)}] + noise^{(k)}(i) \quad (2)$$

where $c \in \mathbb{R}$:constant, $noise^{(k)}$:Gaussian noise .

1.2. Penalties management

In order to protect the aquifer wells from saltwater intrusion, appropriately defined pumping penalties are activated and enforced in each iteration step of the optimization algorithm.

More specifically, Q_{min} and Q_{max} are the penalties controlling respectively the minimum and the maximum volume of water that can be pumped by each well of the aquifer, in order to enforce the second and third constrain of (1) in every iteration (refer to [4] for detailed information).

The $x_{movement}$ penalty, also presented in [4], is responsible for an early attempt of protecting the integrity of each well, by lowering its pumping rate, when saltwater front reaches the $safety_points$ (see subsection 2.4 for a demonstration of this penalty).

For the successful enforcement of the first constrain of (1), the previous penalty is coupled with a new penalty that implements the pumping influence of the i -th well to the j -th well inside the aquifer, in every iteration. For this purpose we introduce the $risk_to_each_other$ $n \times n$ real matrix, every element of which is calculated by the following scheme:

$$\begin{aligned} & \text{if } i \neq j \text{ and } x_{well}(i) \geq x_{well}(j) : \\ & risk_to_each_other^{(k)}(i, j) = \frac{Q_{local}^{max}(i) - Q^{(k)}(i)}{\max\{Q_{local}^{max}\}} * \frac{safety_point(i) - x_{toe}^{(k)}(i)}{L} * \frac{\sqrt{(x_{well}(i) - x_{well}(j))^2 + (y_{well}(i) - y_{well}(j))^2}}{\sqrt{L^2 + W^2}} \\ & \text{else : } risk_to_each_other^{(k)}(i, j) = 0, \text{ for } i, j \in \{1, 2, \dots, n\}, \text{ at } k\text{-th iteration.} \end{aligned} \quad (3)$$

Then, $risk_penalties$ are activated for every aquifer well and applied using the following scheme:

$$\begin{aligned} & \text{if } risk_to_each_other^{(k)}(i, j) = 0 : risk_penalty = 1, \\ & \text{if } risk_to_each_other^{(k)}(i, j) \in (0, 0.020) : risk_penalty = 0.98, \\ & \text{if } risk_to_each_other^{(k)}(i, j) \in [0.020, 0.045) : risk_penalty = 0.96, \\ & \text{if } risk_to_each_other^{(k)}(i, j) \in [0.45, 0.070) : risk_penalty = 0.92, \\ & \text{if } risk_to_each_other^{(k)}(i, j) \in [0.070, 1] : risk_penalty = 0.88. \end{aligned}$$

2. Numerical simulations: Vathi aquifer, Kalymnos, Greece

As in [4] (cf. also[2]), we examine the case of a homogeneous aquifer of almost rectangular geometry, located at the Vathi area on the Greek island Kalymnos, with the following properties: $L = 7000m$, $W = 3000m$, $K = 100m/day$, $q = 1.23m^2/day$, $d = 25m$, $N = 30mm/year$, $safety_distance = 400m$, $Q_{aquifer}^{total} = 20000m^3/day$, $(Q_{local}^{min}, Q_{local}^{max}) = (200, 1500)m^3/day$. Inside the aquifer area there exist 5 pumping wells: $x_{well} = (3932, 2657, 4873, 3353, 4632)m$, $y_{well} = (975, 1572, 1586, 2200, 2470)m$.

The critical well in this area is the well no 2 (see figure 2a), the first one at the left side of the aquifer. This is the closest to the saltwater interface area and so, it is the most likely to be saltwater intruded. In order to prevent this intrusion, every time the saltwater front reaches the $safety_points$ in front of every aquifer well, the pumping penalties presented before are activated to manage the pumping. The objective is to maximize the water pumping from all the aquifer wells, given the constraints. We examine a number of subcases, presenting different pumping setups.

2.1. Vathi aquifer: Main case

The ALOPEX II optimization method, in a typical run of 300 iterations, is called to determine the optimal pumping rates for all aquifer wells, using the following penalty management setup:

$(Q_{min}, Q_{max}) = (1.20, 0.95)$, $x_movement\ penalty=0.95$ and $risk_penalties=(1.00, 0.98, 0.96, 0.92, 0.88)$.

Although the initial values of the pumping rates correspond to a low value of the profit function, the optimization method is able to converge to an optimal solution within a few iterations. The optimal pumping rates achieved during this procedure are the following:

$Q^{opt} = (762.90, 346.21, 1240.82, 1378.89, 206.12)m^3/day$, with $\sum_{i=1}^5 Q^{opt}(i) = 3934.95m^3/day$.

Numbers of penalties activation: $(Q_{min}, Q_{max}, x_movement, risk) = (85, 79, 230, 174)$.

All wells are kept safe from saltwater intrusion (see figure 2a).

2.2. Subcase 1: Optimal pumping rates artificially increased

In order to examine the sensitivity of the solution presented in the main case above, we studied the influence of a 2% increase of the optimal pumping rate of all aquifer wells obtained earlier. The modified new pumping scheme is as follows:

$Q^{mod} = (778.16, 353.13, 1265.64, 1406.48, 210.24)m^3/day$, with $\sum_{i=1}^5 Q^{mod}(i) = 4013.65m^3/day$.

Inspecting figure 2b it becomes obvious that such an increase results to saltwater contamination of well 2. This leads us to the conclusion that the ALOPEX optimization procedure and the penalty management system used in the main case produced near optimal results in the sense that small global perturbations may lead to violation of the constraints.

2.3. Subcase 2: Rain factor N increased

We consider, as an example, the case of a 20% increase on the rain factor N over the aquifer area. The new rain conditions of the aquifer are assumed to be $N = 1.20 * 30 = 36mm/year$. ALOPEX II is called to calculate the new optimal pumping rates. We can see a 7.72% increase of the total volume of pumping water, while all the wells are kept safe from saltwater intrusion. The penalties management setup was the one proposed at the main case (see 2.1) and the new optimal pumping rates are the following:

$Q^{opt} = (1449.21, 240.14, 946.72, 1380.36, 222.27)m^3/day$, with $\sum_{i=1}^5 Q^{opt}(i) = 4238.69m^3/day$.

Numbers of penalties activation: $(Q_{min}, Q_{max}, x_movement, risk) = (60, 114, 215, 148)$.

2.4. Subcase 3: $X_movement$ penalties turned off

The $x_movement$ penalties are activated when the saltwater front reaches the *safety_points* in front of every well (see figure 1b). They are the "first line of defense" against the saltwater advancement to the interior of the aquifer. In this way, when a well is in danger, the first counter-reaction is to reduce the volume of pumping water from the specific well only. This means that the $x_movement$ penalty has only a local effect in the pumping plan of the aquifer. It offers small changes in the total volume of pumping water and as a result, this penalty can only be used for fine tuning of the optimization procedure. On the other hand, lack of this penalty will not destroy the pumping management of the aquifer, but it will result in an erratic behavior of the profit function in the optimization procedure and it is possible that some local extrema will not be examined (see figure 2c). For the optimization run we used the following penalty management setup:

$(Q_{min}, Q_{max}) = (1.20, 0.95)$, $x_movement\ penalty=1.00$ and $risk_penalties=(1.00, 0.98, 0.96, 0.92, 0.88)$.

The optimal values obtained for the pumping rates are the following:

$Q^{opt} = (830.22, 843.38, 732.72, 713.35, 736.74)m^3/day$, with $\sum_{i=1}^5 Q^{opt}(i) = 3856.41m^3/day$.

Numbers of penalties activation: $(Q_{min}, Q_{max}, x_movement, risk) = (165, 177, 0, 187)$.

2.5. Subcase 4: Risk_to_each_other penalties turned off

In order to demonstrate the necessity of the *risk_to_each_other* penalty set and its important role in the optimization process, we considered the case of turning off these penalties. The penalty management setup is as follows:

$(Q_{min}, Q_{max}) = (1.20, 0.95)$, $x_movement\ penalty=0.95$ and $risk_penalties=(1.00, 1.00, 1.00, 1.00, 1.00)$.

The pumping rate values obtained after a few iterations were the following:

$Q = (724.34, 279.59, 1495.30, 292.26, 1449.60)m^3/day$ with $\sum_{i=1}^5 Q(i) = 4241.09m^3/day$

The results are disastrous for the freshwater integrity of the aquifer wells no 2 and 4 (see figure 2d). In a few iterations saltwater contaminates both wells, in spite the fact their pumping rates were dropped close to the minimum acceptable values, due to frequent activation of the $x_movement$ penalty. Wells

no 3 and 5 have almost reached their maximum pumping capabilities, without risking their freshwater integrity, keeping safe at the same time well no 1. The need for a robust pumping management is in this case more obvious than ever.

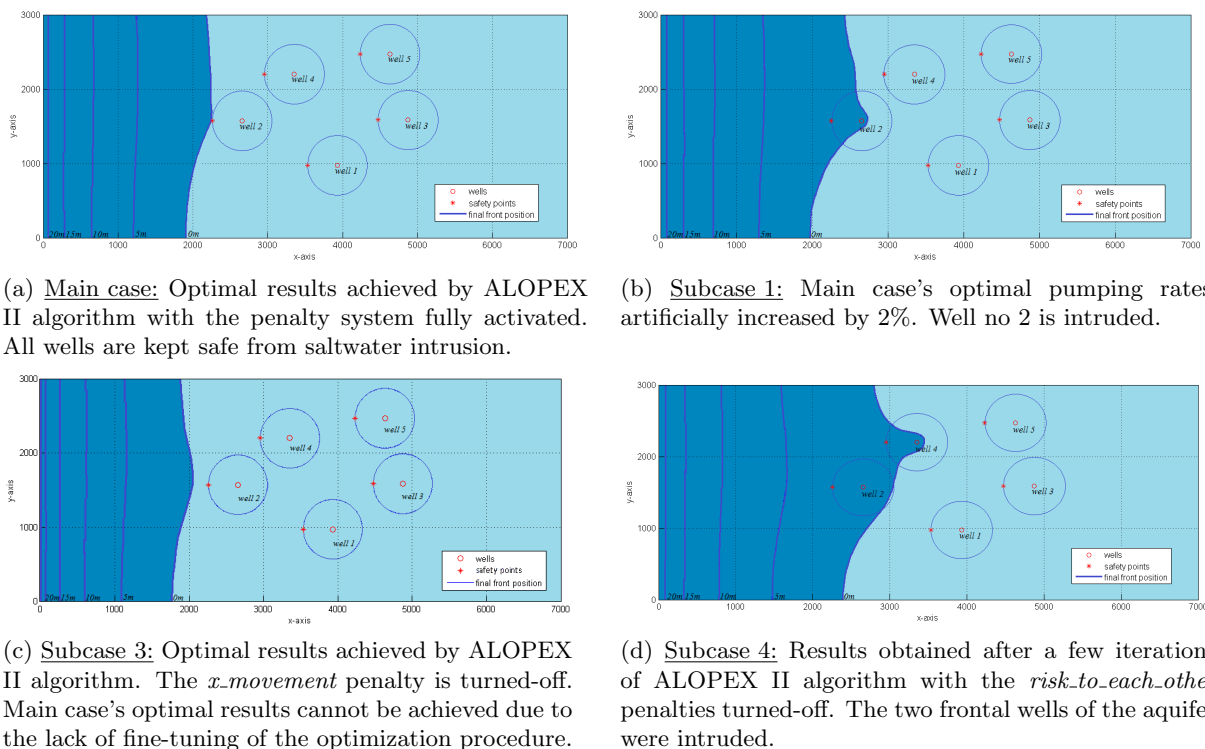


Figure 2: Contour plots of the interface area, at different elevation heights (5m step) from the bottom to the top of the aquifer, after completion of the optimization procedure for the case of Vathi aquifer in the Greek island of Kalymnos.

3. Conclusions

The present work effectively implements the stochastic optimization ALOPEX method, coupled with a constrain management system, to the problem of prevention saltwater intrusion in freshwater aquifers. Simulations are presented for the aquifer of Vathi area, on the Greek island of Kalymnos. A study on the sensitivity of the optimization process confirmed the efficiency and applicability of the optimization method, as well as the importance of the penalties imposed.

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References

- [1] Harth E and Tzanakou E 1974 ALOPEX: a stochastic method for determining visual receptive fields *Vision Research* **14** 1475
- [2] Mantoglou A 2003 Pumping management of coastal aquifers using analytical models of saltwater intrusion *Water Resources Research* **39(12)** 1335
- [3] Strack O D L 1989 *Groundwater Mechanics* (Prentice Hall)
- [4] Stratis P, Saridakis Y, Zakyntinaki M and Papadopoulou E 2013 *Stochastic optimization for a model problem of saltwater intrusion in coastal aquifers* (submitted)
- [5] Zakyntinaki M and Saridakis Y G 2003 Stochastic optimization for a tip-tilt adaptive correcting system *Comp. Phys. Commun.* **150(3)** 274